Industrial Electric Furnaces and Appliances
Volume I

INDUSTRIAL ELECTRIC FURNACES AND APPLIANCES

VOLUME I

By V. PASCHKIS, M.E., E.E., D.Sc.
Research Associate, Heat and Mass Flow Analyzer Laboratory, Department of Mechanical Engineering, Columbia University in the City of New York.

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PREFACE

A survey of the role of electric furnaces in the broad field of industrial heating processes of the metal, ceramic, chemical, and other industries reveals remarkable fluctuations in the past twenty-five years. When compared with the centuries-old practice of melting and heat treatment in fuel-fired equipment, the art of electric heating is relatively young. In its short life span it has witnessed a rapid growth followed by an apparent relative decline.

The industry is again experiencing expansion and is confronted with new problems resulting from recent basic changes in industrial heating practice. These indicate that the designer and user of industrial heating equipment must learn to think more in terms of uniform heat transfer to and from the product to be heated—not merely in comparative terms of electricity or fuel or of temperature control. He must learn that the economic value of a heating operation is expressed in terms of relative uniform quality and over-all cost of the ultimate product. Realistic recognition of these factors and the recent striking development in design of fuel-fired equipment should lead to the constructive development of electric heating devices in the future.

This book is intended as a contribution toward placing on a more rational basis this present expansion of electric heating and toward largely eliminating design by empirical methods. Any one at all familiar with this fascinating field knows that experience of the past will be a guiding factor for many years to come; but it is to be hoped that more accurate knowledge based on sound fundamentals will avoid the errors of the past.

This is true for every branch of electric heating, but particularly in the field of heat transfer which, apparently, has not yet materially influenced the design and operation of many electric furnaces now in service. A frequently encountered misconception always connects heat transfer with fuel economy, at least as far as furnaces are concerned. Actually, uniformity of the heat-treated product is the fundamental thermal consideration and should be accepted as a basis for design and operation of such furnaces and auxiliary cooling equipment.

In this book, special emphasis is placed upon the thermal aspects of furnace design and operation, especially in relation to uniformity of the product to be heated; the frequently unrecognized interrelations of thermal and electrical factors associated with such equipment is stressed. Principles of design are discussed in detail and many types of practical furnaces are illustrated in diagrammatic form. Formulae are included which should prove useful for designers. Because of the rapid changes the patent situation, such references are omitted.

The book is planned in two volumes. In a general chapter, the first volume covers the thermal, electric, and economic principles applying to all types of furnaces and appliances. A second chapter discusses arc furnaces and electrode melting furnaces; here, emphasis is placed on steel melting furnaces. In a separate section the special design of ferro-alloy furnaces is treated. The second volume will cover induction, capacitance, and resistance heating.

It is hoped that the book will be helpful to furnace operators who desire to increase their understanding of the important production tool they use, and that the correlated presentation of basic data may be helpful to the design engineer; also, that it may be helpful to the power engineers, concerned with expansion of the electric heating field, and, particularly, develop in technical and production circles a better understanding of the economic importance of industrial heating—one of the oldest basic arts, influencing the quality and cost of most manufactured products, yet generally misunderstood and poorly practiced throughout the world.

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CHAPTER ONE

Introductory Survey

I. ECONOMIC JUSTIFICATION

One kilowatt-hour (kwhr), the unit of electrical energy, is the equivalent of 3413 Btu. One pound of coal averages 14,000 Btu. One kilowatt-hour costs approximately $0.01 in industrial plants; in the same plants, one pound of coal costs $0.003. (The cost of electric energy as well as that of coal holds for medium or small plants. Large consumers pay less for either commodity.) Hence, 1000 Btu produced electrically costs $0.003 and 1000 Btu produced from coal costs $0.0002. Consider also a comparative flowsheet of energy for an electric furnace and a fuel-fired furnace (Fig. 1). The flowsheet is drawn for electrical energy produced from a steam power plant. In the case of electrically generated heat, energy undergoes four conversions (two conversions are saved in hydroelectric installations); in the case of fuel-fired furnaces, there is only one conversion. Each energy conversion is subject to losses: in the electric furnace there are four sources of energy loss and in the fuel-fired furnace, only one.

Why, then, is electrical heat used? Why are electric furnaces employed?

An electric furnace is often more economical than a fuel-fired furnace for two general reasons: (*1*) the relatively high efficiency of electric heating which often makes it less expensive than heating by fuel; (*2*) incidental advantages—such as ease of control, cleanliness, etc.

The efficiency of conversion of fuel into useful heat in small units is low, especially in industrial furnaces. The efficiency of large steam generators such as are used in central power stations is very high. On a percentage basis, decentralized combustion in fuel-fired furnaces results in higher losses than centralized transformation in large power plants. The electric furnace is, in most instances, a very desirable load for power companies because of the good power factor and relatively high load factor. Operators of electric furnaces are, therefore, often offered a better rate than the average consumer. Furthermore, the electric furnace can often be used to smooth out depressions in the power station load diagram. A typical load diagram showing how the power demand varies

with different hours of the day is illustrated by line *A* of Figure 2. At midnight the load has a fairly high value, due to contributions from street lighting, theaters, etc. Toward morning the load drops, and then increases rapidly with the start of operation in manufacturing plants. Usually the morning peak is followed by a gradual decrease, as street cars run less frequently and absorb less power. At the noon session there is a pronounced dip followed by a slow build-up toward an evening peak.

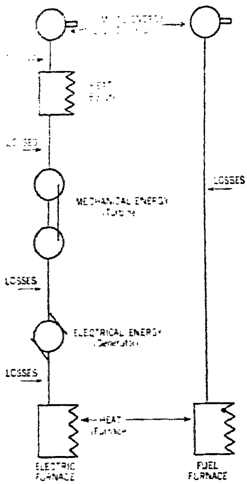

FIG. 1.—Flow of energy to a fuel-fired furnace and an electric furnace.

If electric furnaces are operated 24 hours per day and connected to the system, they can be operated without regard to the power system. Such operation could be represented in the diagram by lowering the abscissa axis according to the added constant power of the furnaces. It is, however, also possible to utilize the furnaces to even out the demand curve; during the peak hours the furnaces might just run idle or with very small load. During the slump hours of the demand curve they would operate at their maximum load. Thus it is possible to obtain a straight line for the demand curve, as shown (dotted) in Figure 2. Depending on the size of the power plant, a greater or smaller number of furnaces would have to contribute to obtaining a straight-line power demand as shown in the illustration. But even if no such complete uniformity can be reached, the operation of furnaces can result in an improved demand curve.

Assuming that sufficient furnaces are working according to such a schedule as to yield the demand curve *B*, the maximum demand increases only slightly; but the consumption (shaded area) gains considerably. In most power rates, the fixed and the proportional costs of generating power are reflected directly or indirectly. (The fixed cost includes depreciation, interest on capital, and administration; the proportional cost includes fuel and maintenance material and part of the personnel charges.) Progressive power companies pass on the savings from improvements in the demand curve to the customers responsible for such improvement. Consequently, the power charged for the furnace will be lower than the average. In many cases, electric furnaces can be run in such a manner as to smooth out the peaks in central power station loads and take advantage of low power rates. See page 70.

There are many periods when a furnace runs "idle." In most cases, the cost of idling is smaller for an electric furnace than that for a fuel-fired furnace. The word "idling" is used here in the sense that the furnace is kept at an elevated temperature without productive use of heat. "Idling losses" as used here may be defined as the total energy losses incurred in a furnace after thermal equilibrium has been reached,

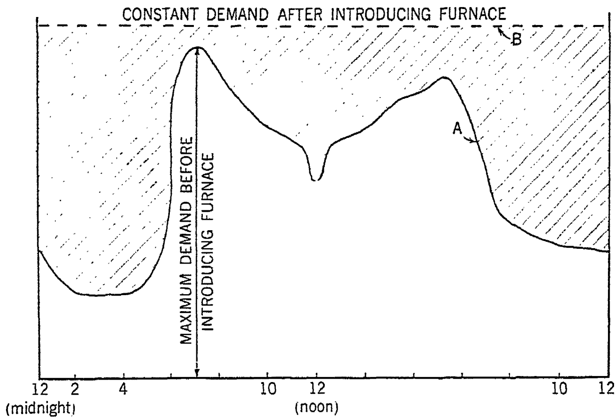

FIG. 2.—Load diagram, illustrating morning peak when factory starts. The diagram changes with season and with productivity of industry. Shaded area, furnace power consumption.

at which point no more energy is absorbed by the charge. Idling can occur either in waiting for the next charge or in maintaining the charge in the furnace at a constant elevated temperature after the elevated temperature has been reached. Examples of the latter can be seen in the refining period in arc furnaces, in which the charge is kept at an elevated temperature for a considerable time after the melting point has been reached, or in holding a case hardening load at high temperature in order to obtain sufficiently deep carburization. Consequently, a process calling for considerable idling, resulting in furnaces of relatively small output per unit (*e. g.*, unit of hearth area) will be more suitable for electric furnaces than a process with a very short heating cycle and small idling losses. There are exceptions to this, *e. g.*, induction hardening. (For further details, see page 68.)

In some heating operations, electric heating is the most desirable way—in other instances, the only way—in which heat can be applied. An example of this is induction hardening, in which heat is generated in the piece itself at exactly the place at which it is wanted—the surface. In comparison, flame hardening, which is used to produce results similar to induction hardening, is much more complicated and is almost like an

artificial substitute. Another example, one in which the incidental advantages are not quite so pronounced, is the electrode salt bath furnace, the salt being used for more uniform heating and frequently for metallurgical reasons. Here the generation of heat in the salt, using the salt as resistor, is much simpler than heating the salt through the wall of the pot from the outside, as is done in the fuel-fired salt bath furnaces.

Electric heat frequently involves advantages such as simplicity and accuracy of control, particularly for melting furnaces, cleanliness, etc., which outweigh the possibly higher cost of heat. These advantages will be discussed later (page 13). A word of caution, however, will not be amiss: these advantages, although existing in many instances, are often abused as selling points in cases to which they do not properly apply.

II. FURNACE TYPES

The most appropriate classification of electric furnaces is based on their electrical characteristics, principally the manner in which electrical energy is converted into heat.[1] There are two such types of conversion: in the arc; and in resistors—solid or liquid. Electric furnaces are therefore divided into arc-type and resistance furnaces. These in turn may be divided into two groups: direct-heat furnaces and indirect-heat furnaces, the charge being part of the electric circuit in the former and not part of the electric heating circuit in the latter.

Direct-heat resistance furnaces can be grouped into electrode resistance furnaces and induction furnaces. In electrode resistance furnaces, the current is applied to the charge by means of electrodes; in induction furnaces, the current is transmitted without physical contact between the supply system and the charge. In electrode resistance furnaces operating at high frequencies, the electrode need not touch the charge: the high-frequency current can flow from the electrode through an air gap to the charge. In electrode resistance furnaces for normal commercial frequencies, physical contact between electrode and charge is necessary. Some induction furnaces have magnetic cores; others operate without cores. In indirect-heat resistance furnaces, the charge is by definition not part of the heating circuit; separate resistors are used. Such furnaces will be called hereafter resistor furnaces.

The classification of electric furnaces can be schematically shown as follows (see also Fig. 3):

A. *Arc furnaces*

1. Direct-heat (A_1)
2. Indirect-heat (A_2)

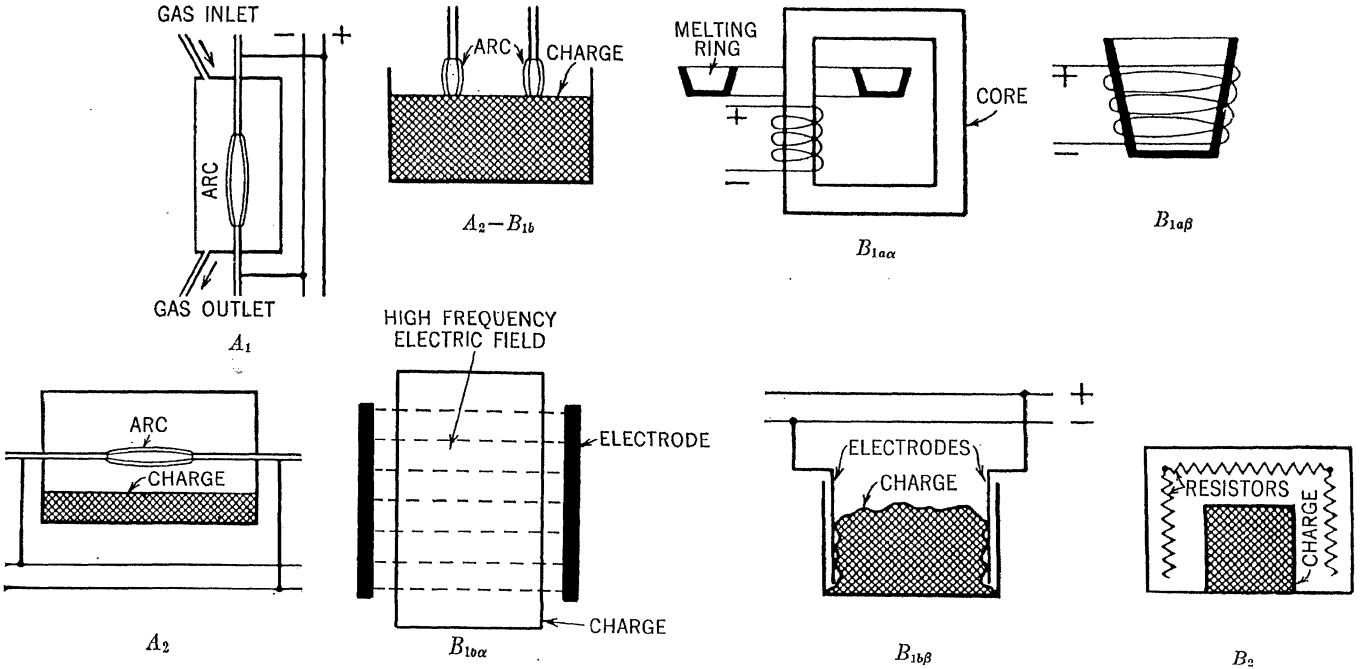

FIG. 3.—Furnace types. A_1, direct-heat arc furnace. A_2, indirect-heat arc furnace. A_2–B_{1b}, combined type: indirect-heat arc and direct-heat low-frequency resistor furnace (current flows through the charge from the tip of one electrode to the other). B_{1a}, direct-heat resistance furnace (energy transfer by induction: α, with core; β, without core. B_{1b}, direct-heat resistance furnace (energy transfer by electrodes): α, high-frequency (capacitance); β, low-frequency ("electrode furnace"). B_2, indirect-heat resistance furnace (heat transfer from the resistors to the charge by radiation and/or convection).

B. *Resistance furnaces*
  1. Direct-heat (B_1)
    a. Induction
      α. With core ($B_{1a\alpha}$)
      β. Coreless [1] ($B_{1a\beta}$)
    b. Electrode
      α. High-frequency capacitance [2] ($B_{1b\alpha}$)
      β. Low-frequency ($B_{1b\beta}$)
  2. Indirect-heat (B_2)

Obviously there are combinations of the various types, the most common being a combination of an indirect-heat arc furnace and a direct-heat resistance furnace (A_2–$B_{1b\beta}$).

It is customary to classify induction furnaces as a separate group. There will then be three main types: arc, induction, and resistance. Although this classification is less exact than the one just given, it will be adopted for the sake of convenience in subdividing the book. The HFC furnace will be included in the chapter on induction furnaces because of the similarity of the power supply for the high-frequency induction and the capacitance-type heating furnaces.

III. USES

Electric furnaces have found innumerable applications in industry. The compilation of a complete list is impractical, if not impossible; in addition, the list would be far too lengthy for inclusion in this book. Table I contains only the most commonly known applications along with the furnace type used. The furnace type, or types, employed for each case is indicated by the asterisk. As will be noted, in a number of instances, two types of furnace can be used for one purpose.

IV. SELECTION OF FURNACE TYPES

The selection of the type of furnace best suited for a particular purpose is based on the following considerations. Arc furnaces are limited in temperature by the temperature of the arc, which is approximately 6600 F. Direct-heat resistance furnaces—*i. e.*, induction furnaces and electrode furnaces—are limited in temperature only by the refractoriness of the wall material and electrodes, not by any limit in the temperature of

[1] This type of furnace will henceforth be designated as an HFI furnace—meaning high-frequency induction furnace—because coreless induction furnaces are operated at high frequencies.

[2] This type of furnace will henceforth be designated as an HFC furnace, which stands for high-frequency capacitance furnace.

TABLE I

ELECTRIC FURNACE APPLICATIONS

Application	Arc	Resistance			
		Direct-heat			Indirect-heat
		Induction	Electrode		
			HFC	Low-frequency	
I. METALS					
Aluminum:					
production from bauxite				[a]	
melting for refining and casting		*			*
heating, annealing, and heat treating					*
Brass and bronze:					
melting for refining	*	*			*[b]
melting for casting		*			*
heating, annealing, and heat treating		*			*
Copper:					
melting for refining	*	*			*[b]
heating, annealing, and heat treating		*			*
Iron and Steel:					
production from ore	*[c]			*[c]	
production of ferro-alloys	*[c]			*[c]	
melting for refining and casting	*	*			*[b]
heating and heat treating (annealing, hardening, etc.)		*			*
Zinc:					
production	*			*	
melting for refining and casting		*			*
melting for galvanizing					*
II. OTHER INORGANIC MATERIALS					
Carbides:					
Calcium carbide	*[c]			*[c]	
Silicon carbide				*	
Ceramics and glass:					
melting				*	*[d]
heating, heat treating and cooling			*		*
Nitrogen and nitrates	*				
Phosphates	*[c]			*[c]	
Graphite				*	
Paints (drying)					*
III. ORGANIC MATERIALS					
Chemical industrial processes	*	*	*	*	*
Foodstuffs:					
drying, sterilizing		*[e]	*		*[c]
Wood		*	*		*
Synthetic resins			*		*

[a] This is an electrolytic process operating with electrode at elevated temperatures; direct current must be used.

[b] Application of this furnace type rare; best known type is the George furnace, a French development.

[c] Combined arc–resistance furnace.

[d] Only for small units.

[e] Electric heat rarely applied in the United States.

the heat source; if suitable refractory material were available, any temperature could be obtained; these furnaces can work at very low as well as extremely high temperatures. Indirect-heat resistance furnaces are limited in temperature to the allowable resistor temperature; with very few exceptions, no resistors are available today which operate at temperatures higher than 2700 F. High-frequency capacitance furnaces (HFC furnaces) are suited only for materials which are relatively poor electric conductors.

From these considerations the following general rule can be stated: For processes calling for temperatures of 2400 F or higher, arc or direct-heat resistor furnaces (including induction furnaces) are used[3]; for processes calling for temperatures lower than 2400 F, direct- or indirect-heat resistor furnaces are used.

Low-frequency electrode resistance furnaces are quite inexpensive to build if the charge consists of material of fairly high resistance. This type of furnace is therefore generally applied to so-called semiconductors, carbon, graphite, silicon carbide, and molten glass. It is not applied to the melting of metals.

From the foregoing it is obvious that arc furnaces and indirect-heat resistance furnaces are not competing types. The three competing types are: arc and induction for high temperatures; induction and indirect resistance for low temperatures; and, finally, high-frequency capacitance heating and indirect-heat resistance furnaces for nonmetallic materials.

A. Arc and Induction Furnaces

Before discussing the relative merits of these two types, the field of competition between them can be considerably narrowed down by the limitations of size and of temperature.

In the United States, induction furnaces have heretofore not been built for a capacity larger than four tons. (In Europe, a 10-ton furnace is said to be in operation.) Arc furnaces for steel melting have been built for a maximum capacity of 100 tons. In comparing these figures it must be kept in mind, however, that the speed of melting in the induction furnace in the range of about 100 lb to two tons is approximately twice that of the arc furnace. From this it follows that electric furnaces for large output must invariably be of the arc type. In regard to temperature, melting operations in the lower part of the temperature range in question (*i. e.*, above 2400 F) are limited, in some cases, to induction furnaces because of the undesirable influence of the high arc temperatures. The melting of brass, for instance, is carried out almost entirely in core-

[3] Exceptions might be mentioned, for example the indirect-heat resistor furnace for the melting of iron and steel. H. George, *Trans. Am. Electrochem. Soc.*, 68, 53 (1935).

type induction furnaces because the high temperatures of the arc would tend to burn the zinc out of the brass. The debatable field of operations includes the refining of steel, and the melting of nickel and similar metals having a high melting temperature.

A comparison between the arc and the high-frequency furnace (HFI) for melting steel yields the following picture:

(1) *Output.* For the same volume of melting space, the coreless induction furnace generally has an output twice that of an arc furnace, in sizes between 100 lb and two tons. For sizes outside this range, both arc and HFI furnaces have approximately the same output per unit of bath volume. The higher output in the indicated range is due to the fact that HFI furnaces of these sizes are usually built for twice the connected load customarily applied to arc furnaces of the same size.

(2) *Temperature uniformity.* Local overheating in the immediate neighborhood of the arc is unavoidable. No overheating occurs in the HFI furnace.

(3) *Bath movement.* Because of the electromagnetic forces in the HFI furnace, the bath of molten metal is continually in motion. In the arc furnace, the metal is hardly moved automatically, and hand stirring is therefore necessary to obtain mixing. (The rocking arc furnaces are relatively limited in output). When mixing is undesirable, the arc furnace is preferred.

(4) *Slag temperature.* In the arc furnace, the slag has the same or even a slightly higher temperature than the bath. In the HFI furnace, the slag is heated only indirectly by contact with the bath and is therefore always colder than the bath.

(5) *Power consumption.* In continuous operation, arc and HFI furnaces of the same output have approximately the same power consumption per ton. For intermittent operation, however, the HFI furnace is decidedly superior because heat storage in the furnace structure is only a fraction of that in the arc furnace.

(6) *Lining.* The wall of the container holding the melt is necessarily very thin in the HFI furnace and generally quite thick in the arc furnace. In order to avoid leakage, the lining in the HFI furnace must be monolithic. Thus, there is a limitation to the choice of the wall material in the HFI furnace. There is no such limitation in the arc furnace.

(7) *Metallurgical results.* Careful examinations have shown that material of equal quality can be obtained from both types of furnace.[4] No specific rules as to the most desirable fields of application for either type

[4] E. Houdremont, H. Kallen, and K. Gebhard, *Stahl u. Eisen,* 55, 228 (1935). R. Scherer, *ibid.,* 55, 276 (1935).

can be given. Although the HFI furnace is generally not used for refining, it has been used in some instances for this purpose, the bath being quieted by switching the power off temporarily after overheating the charge.

(8) *Floor space.* A four-ton (holding capacity) furnace requires, as an arc furnace, approximately 700 sq ft or 175 sq ft per ton of holding capacity, and, as a high-frequency furnace, approximately 2000 sq ft or 500 sq ft per ton of holding capacity.

(9) *Price.* The price of furnaces depends largely on the connected load, available voltage, etc. HFI furnaces are in general higher in first cost than are arc furnaces, even if the higher speed of melting is considered. By way of illustration, a 1250-kw, four-ton HFI furnace would cost approximately $25,000 to $30,000 per ton, while an arc furnace similarly powered and of equal size would cost about $12,500 to $15,000 per ton.

(10) *Cost of operation.* Since the power consumption is the same for both arc and HFI furnace, the comparison of their cost of operation will depend on other items. The cost of the lining depends on a very large number of factors which differ from furnace to furnace. The cost for the arc furnace is approximately one-half that for the HFI furnace, being about $0.65 per ton of melted steel in the basic arc furnace and $1.25 per ton in the induction furnace with basic lining. With acid lining, there is almost no difference between the cost of lining for the two types. In the arc furnace, the acid lining costs, per ton, approximately 50% of the basic lining, while the acid lining in the HFI costs approximately 25% of the basic lining.

(11) *Water cooling.* The water consumption in small furnaces does not differ much for the two types. In small sizes, where the HFI furnace has a decided advantage in melting time, the HFI furnace uses less water, but in large sizes, *e. g.*, four tons, it uses about twice the amount of water consumed in the arc furnace.

B. Induction and Resistor Furnaces

Furnaces for melting and for heating will be considered separately.

1. Melting

The induction furnace has a violent bath movement: the resistor furnace has little or no bath movement. For some metals, such as aluminum, a stirring action is undesirable, because it results in oxidation. For other metals, such as light alloys, a stirring action may be beneficial, for the automatic stirring effects a good mixing of the components. The speed of melting is much higher in the induction furnace than in the

resistor furnace. The size of induction furnaces is limited in comparison with the largest sizes of resistor furnaces. The ratio of free surface of the bath to the volume of molten metal subject to oxidation is much larger in resistor furnaces than in induction furnaces.

2. Heating

Induction heating is as yet limited to metallic charges. Attempts to use high-frequency induction heat for firing ceramic materials and for drying purposes have been made but have not reached the stage of industrial application. Heat generation in induction heating is limited to a relatively thin skin or layer of the piece to be treated. Inductive heat treatment can be considered as competing with resistance heating in cases in which the time for transferring heat from the resistor to the surface of the charge is a major part of the total heating time. When the time to equalize the heat within the charge is appreciable, however, induction heat must generally be ruled out, because in order to obtain high electric efficiency no, or almost no, thermal insulation can be applied. In other words, inductive heat treating is at present limited to operations in which each heat does not require more than a few seconds.

The generation of heat in a thin layer is, on the other hand, much more rapid in the induction heater than in a resistor furnace. Inductive heat can be applied more easily locally than any other method of heating. Inductive heating of the neck of cartridges is a good example.[5]

In order to carry out inductive heat treatment, special coils are necessary for each type of piece to be treated. For each shape and size, the correct heating process (time, power etc.) must be determined by trial-and-error methods. These conditions make inductive heat treatment worth while only if a large number of identical pieces of the same size, shape, and material are to be treated. For small production, indirect resistance heating is indicated.

C. High-Frequency Capacitance and Indirect-Heat Resistor Furnaces

High-frequency capacitance (HFC) heating is applied only to materials which are poor electric conductors. Such materials are also poor thermal conductors. In the indirect-heat resistor furnace, the material is heated entirely from the surface. Temperature differences in the load are high during the initial stage of heating but gradually level off during the final stage. In HFC furnaces, on the other hand, heat is generated through the entire body at a uniform rate, and temperature differences occur only at the surface, where the charge is cooled by the surroundings.

[5] M. Viry, *Bull. soc. franç. élec.*, 10, 416 (1930). F. T. Chesnut, *Mech. Eng.*, 63, 861 (1941).

Conditions change if the body is not uniform in material or in cross section. Then heat generation also becomes nonuniform, and the thermal superiority of the HFC furnaces no longer exists. Thus the HFC furnace is definitely superior in thermal aspect for pieces of uniform shape and

TABLE II

SELECTION OF FURNACE TYPES

Factors	Arc	Resistance				
		Direct-heat				Indirect-heat
		Induction		Electrode		
		HFI	Low-frequency	HFC	Low-frequency	
Temperature limit	6600 F	Only through furnace lining				2700 F
Lines of competition:						
melting	+	+	+		+	
heat treating metals		+	(+)			+
heat treating nonconductors				+		+
Advantages:						
melting	Heated slag brings great freedom in selection of slag	High melting speed. Stirring of metal by electric force. Freedom in selection of lining material				
heat treating metals		Rapid heating of a thin layer				High thermal efficiency. Furnace independent of shape of piece
heat treating nonconductors				Heat generated within the body		Independent of shape. Great flexibility

material. However similar to HFI heat treating, special electrodes must be applied for each shape to be heated, and power and duration of application must be determined for each piece.

HFC furnaces, because of the high-frequency power supply, are probably more expensive than indirect-heat resistor furnaces. HFC heating is therefore usually limited to mass production problems.

The preceding considerations are summarized in Table II.

V. ADVANTAGES AND DISADVANTAGES

When electric furnaces were first introduced into industry (arc furnaces in 1906, resistor furnaces from 1920 to 1922), too much was frequently expected of them: they were believed to be a cure-all. After this period of overexpectation came a period of skepticism. And since the discussion "for" or "against" is still somewhat heated, in order to obtain an objective picture it is worth while considering briefly the circumstances involved. It is difficult to formulate general statements without becoming lost in generalities, but the question is of such importance that an attempt must be made at least to weigh the advantages against the disadvantages.

A. Over-all Considerations

1. Advantages

Reproducibility.—In electric furnaces it is extremely easy to repeat any cycle which has once been established as the most desirable. Once the settings are made on the controls, nothing further is necessary in order always to obtain the same results provided the charge is identical and always arranged in the same way. Reproducibility of any process, especially of a heat process, tends to improve the quality of the product.

Ease of Metering.—It is highly desirable to carry out measurements of furnace operations. Such measurements generally consist of temperature determinations and measurements of heat quantities. Temperature measurements offer the same difficulties in electric furnaces as in fuel-fired units. The measurement of heat quantities, however, offers practically no difficulty in the electric furnace because a watt-hour meter supplies all the necessary information. In the fuel-fired furnace, no direct measurement is possible: at best, fuel quantities can be measured, that is, weight of coal or coke, amount of gas or oil, etc., and even then there is still always doubt as to the caloric value of the fuel. Frequent analysis of the fuel in order to obtain this value is necessary.

The ease with which heat determinations can be made on electric furnaces is of considerable advantage. In order to make full use of it, it is advisable to keep records of the kwhr used. The recording should be done at regular intervals. Together with notations concerning weight of load and time of cycles and possibly the temperatures, such records allow immediate detection of irregularities. For example, an increase in power consumption could indicate incorrect operation involving too long a period of open door or too long a time for charging; decrease of power consumption would indicate too short a time cycle with consequent nonuniformity of temperature. Difficulties and trends calling for early repair, etc., can thus be found.

2. Disadvantages

Fuel Storage.—The heat supply for fuel-fired furnaces can be stored in the plant, thus making the furnace independent for some time of outside disturbances such as strikes, transportation difficulties, etc. (Obviously, so long as the power group functions, the electric furnace is ready to operate any time.) If, however, it is desired to take advantage of fuel storage, the cost of the storage (interest, space for the storeroom, etc.) must be considered. For cases in which reserve storage is unnecessary, this disadvantage of course no longer applies. Because of the saving in storage charges, the electric furnace has an advantage over the coal- or oil-fired furnace.

First Cost.—In almost every case, the electric furnace is higher in price than a fuel-fired furnace of the same output. This is especially true of heat-treating furnaces. Arc furnaces for melting compare favorably with open-hearth furnaces insofar as first cost is concerned.

B. Melting Furnaces

Furnace Atmosphere.—Because no combustion takes place in the electric furnace there is no movement of gases over the bath and hence almost no danger of contaminating the metal with gases. (In some cases the contact of metal with the combustion gases is not objectionable; then this advantage for the electric furnace drops out.) Vacuum furnaces are almost entirely limited to electric heat.

Higher Temperatures.—Higher temperatures may be obtained in electric furnaces than in fuel-fired units. The temperature obtainable in induction heat (not considering limitations due to furnace material but only such as are inherent to the source of heat) is unlimited: the temperature of the electric arc is approximately 6600 F, whereas the hottest burning fuels with customary preheat of air yield only a temperature of approximately 3900 F. Obviously, in any heating process only the temperature gradient from the source of heat to the process temperature can be utilized. Heat, even in unlimited quantities, at a lower temperature level than that of the process is useless except for preheating. In this connection, the curve[6] in Fig. 4 is of interest. Here, the heat contents of various fuels are plotted as abscissas (Btu per lb or Btu per cu ft). The figure does not include the latent chemical energy, but only the energy available in the combustion gases under ideal combustion conditions without preheat of air. The highest obtainable temperatures are plotted as ordinates. For the sake of completeness the limits for resistance heating, induction heat (limited by design materials) and arc heating

[6] J. Wotschke, *Grundlagen des elektrischen Schmelzofens*. Knapp, Halle, 1933, p. 466.

are also included. For any process, draw a horizontal line at the temperature necessary for the process. The difference of temperature between this horizontal line and that prevailing for any fuel indicates the available temperature difference. For instance, for a temperature of 3550 F, the available gradient with oil with no preheat of air will be at best only 350 F.

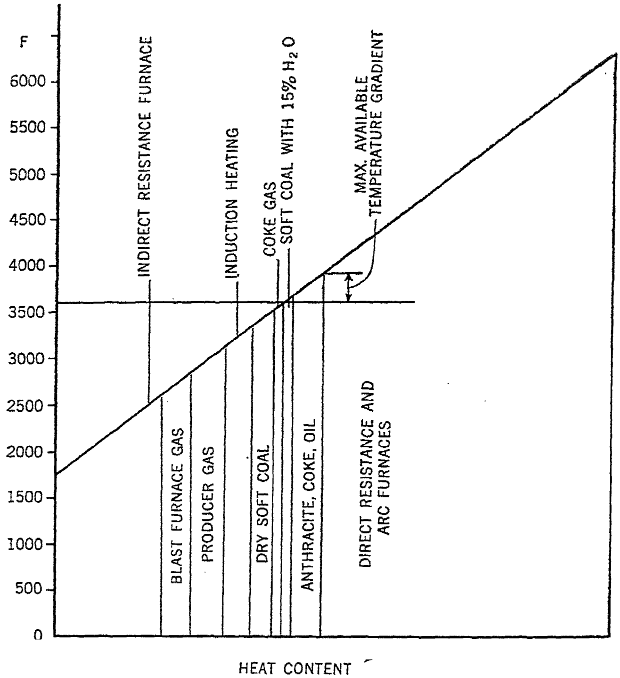

FIG. 4.—Heat content *vs.* temperature.

1. Advantages

Electric melting furnaces are in general built *in smaller units* than fuel-fired furnaces. The smaller size results in better metallurgical control and greater ease in changing the composition of the melt: this refers to addition of small amounts of alloying elements as well as to a complete change of composition.

Construction.—Because of the small temperature gradient, fuel-fired melting furnaces require large quantities of fuel, which yield a high amount of waste gases. The heat contained in these gases (at too low a temperature level) must be utilized, and this calls for regenerators, recuperators, and similar equipment requiring large space, frequently built underneath the furnace proper. The use of flue channels and stack make the construction of fuel-fired furnaces very much more complicated than that of electric melting furnaces.

Operation.—The ease with which the heat input is changed in an electric furnace makes its operation easier and more flexible than that of a fuel-fired unit.

Space.—The higher temperature gradient available in the electric arc and induction melting furnace results in quicker melting. Such an electric furnace of given holding capacity yields a higher tonnage than a fuel-fired furnace of the same holding capacity; hence the necessary area per ton output is smaller for the electric furnace than for the fuel-fired unit.

2. Disadvantages

Melting Speed.—The gases flowing over the metal and in direct contact with the charge result, in some instances, in quicker melting than is possible in electric furnaces. This is especially true at lower temperatures (*e. g.*, for aluminum).

Maintenance.—Electric furnaces usually require a refractory lining of higher quality and cost than do fuel-fired units, mainly because of the higher temperatures at which electric furnaces are operated. In many instances the higher price of the lining is offset by its longer life.

Operation.—The electric furnace calls for electricians besides the men skilled in the metallurgical process of melting. (Such electricians are sometimes not available. The savings due to their absence is then mostly lost by the higher cost of other items.) Because of this and the high cost of heat in the form of electrical energy, the operating cost of electric melting furnaces (disregarding questions of quality) is higher than that of fuel-fired furnaces.

C. Heat-Treating Furnaces

Advantages and disadvantages can be considered here only for the average furnace. Special designs, as for electrode salt bath furnaces, etc., may have additional features which change the following picture.

1. Advantages

Temperature Uniformity.—It is customary to ascribe to the electric (heat-treating) furnace better temperature uniformity than to a fuel-fired unit. Before examining this claim it should be understood that there are two types of temperature uniformity, both of which, although entirely unrelated, influence the uniformity of temperature in the charge. These two types are: uniformity in space and uniformity with time. A furnace can be so designed that, at any given time, the temperatures over the entire working space are the same (this refers to the empty furnace); but, as the heat input changes, this temperature, uniform in space, changes with time. The electric resistor furnace is much superior to the fuel-fired furnace as far as temperature uniformity in space is concerned:

the heat can be either uniformly generated or so generated that the temperature in the (empty) furnace is uniform, or, finally, so that heat transfer to the charge is uniform; in the fuel-fired furnace such distribution is much harder to achieve, if it can be achieved at all. Since automatic control of temperature has been developed for electric and gas furnaces, the temperature uniformity in time is not always better in the electric furnace than in the fuel-fired unit: for the same load in the furnace there are two elements influencing the uniformity of temperature with time—the nature of the inside lining of the furnace and the ratio of heat input to instantaneous heat consumption. The properties of the inside lining are in many instances more suitable for accurate temperature control in electric furnaces than in fuel-fired units. The ratio of heat input to instantaneous heat consumption, however, can be much more easily adjusted in fuel-fired furnaces, by setting the valve, than in electric furnaces, in which relatively complicated controls must be used. Finally, it must not be forgotten that uniformity in time and space do not of themselves mean that there is uniformity of temperature within the charge. In the final analysis, the nature and properties of the charge, its arrangement in the furnace, the furnace design itself, and many other factors must also be taken into account.

Atmospheric Control.—Since there is no combustion in the electric furnace, it is possible by injecting gas to maintain any desired atmosphere in it. In the fuel-fired type, freedom of choice of atmosphere is possible only for indirect-heat furnaces, in which the fuel is burnt outside a muffle or inside radiant tubes, etc. But it should be noted that all these means of indirect heat entail other difficulties. In direct-heat fuel-fired furnaces, a change of temperature also changes the amount of fuel. The atmosphere may change if no special precautions are taken.

Human Element.—From the viewpoint of control, the operation of electric furnaces can be made almost entirely independent of the skill and attentiveness of the operator, especially the heating, which is carried out entirely automatically. Mechanical feeding of solid fuel and temperature control tend to make the fuel-fired furnace now more independent of the human element than it was previously; but no fuel (gas, oil, coal) is always of the same composition. The change in composition calls for different settings of atmosphere control and, for best fuel economy, of the air–fuel ratio. These changes cannot as yet be carried out automatically.

Space.—Although generalized statements are dangerous, it can be said that the electric furnace takes up less shop area than the fuel-fired furnace. In addition, other factors such as storage of fuel, disposal of cinders, etc., have to be taken into account.

2. Disadvantages

Repair.—As a rule, electric furnaces are more difficult to repair than fuel-fired furnaces, and require more skilled maintenance crews.

Forcing.—Increasing the output temporarily at the expense of uniformity of output is more limited in electric furnaces than in fuel-fired furnaces. Forcing generally means increase in the temperature of the furnace; and such an increase is more injurious to an electric furnace than to a fuel-fired furnace.

Maintenance.—In order to produce good results, electric furnaces demand a certain amount of care, *e. g.*, the accumulation of dirt and dross (oxidation) on the hearth of resistor furnaces must be avoided, especially when bottom heat is applied; otherwise the bottom resistors may burn out. In general, the consequences of neglect in the maintenance of an electric furnace are more serious than in a fuel-fired furnace.

D. Conclusion

A survey of the advantages and disadvantages of electric furnaces is given in Table III.

TABLE III

ADVANTAGES AND DISADVANTAGES OF ELECTRIC FURNACES

Type	Advantages	Disadvantages
General	Reproducibility of charges. Easy metering	No possibility of fuel storage. Higher first cost
Melting	Smaller units. Better metallurgical control. Furnace atmosphere selected. Higher temperatures obtainable. Simpler construction. Simplified operation. Smaller space required	Skilled maintenance required. Better refractories needed. Electrical operator needed
Heat-treating	Uniformity in space better. Atmospheric control possible. Human element eliminated. Smaller space required	Uniformity in time harder to achieve. More expensive to repair. Forcing the furnace impossible. Sensitivity to abuse

VI. FUNDAMENTALS OF FURNACE CALCULATIONS

A. Thermal

1. Theoretical Basis

The basic function of an industrial furnace is to raise as uniformly as possible the temperature of the charge to a predetermined level which in all instances is higher than that of the surroundings. Sometimes another function may be imposed—maintaining a charge at a predetermined temperature level. A furnace may perform both functions or only one. A soaking pit is an example of the latter case. Hot ingots, which are

already at an elevated temperature, are placed in the pit and kept there until a desired degree of uniformity in temperature throughout the ingots has been achieved.

The thermal part of furnace calculations is thus concerned with temperature differences. Temperature differences are always accompanied by a transmission of heat, since there are no perfect thermal insulators. The laws of heat transmission are of final importance in the design of the many diverse forms of industrial furnaces and other heating equipment.

Heat flow is generally classified on the basis of the three distinct mechanisms by which it is governed 1) conduction, 2) convection, and 3) radiation. It can also be classified on the basis of time dependency as steady or transient. 1) *Conduction* occurs if the heat flow takes place entirely within a solid body (in liquids a combination of conduction and convection occurs); conduction also occurs between two solids if they are in very intimate contact. *Convection* is a form of heat transfer which takes place together with mass transfer and is therefore limited to fluids and gases; as particles of the fluid or gas move from hotter to colder parts they convey the heat contained in each particle. *Radiation* takes place between solids or a solid and a liquid separated by gas or vacuum; the energy is exchanged by means of heat waves without heating of the intermediate gas; compact masses of gas can act as a solid body as far as radiation is concerned; the radiation of heat by furnace gases plays an important role in heat transfer in fuel-fired furnaces. *Steady state* prevails if the temperature at no point in the system changes with time. *Transient state* prevails if the temperature at any point in the system changes with the time.

(a) *The Steady State*

Although the transient state occurs in all furnace operations, the calculations for this state are so complex that it will be well to consider first the steady state.

CONDUCTION

Consider a wall having parallel surfaces, the wall being of finite thickness, L, but infinite in the other two directions (Fig. 5). The heat flow, Q_L, through area A of the wall shall be determined. The wall is subject to a temperature difference, $t_{si} - t_{so}$, t_{si} designating the temperature of the inner surface and t_{so}, the temperature of the outer surface. Designate the thermal conductivity of the wall as k. (Values of k for different materials are given in the appendix.) If L is in feet, the dimensions of k are Btu per ft, hr, F. It is the amount of heat flowing through the square foot of a body of one foot thickness, if the body is subjected to a temperature difference of one degree Fahrenheit. Therefore con-

ductivity is often expressed in Btu per sq ft, hr, F per ft or, written differently, Btu × ft per sq ft, hr, F. These expressions can be converted into the simple form of Btu per ft, hr, F, which is being used here. From the basic equation for thermal conduction in the steady state:

$$Q_L = (t_{si} - t_{so}) \frac{k}{L} \cdot A \tag{1}$$

It should be noted that this equation is identical with Ohm's law governing the flow of electricity in a conductor (see page 59). If the heat path

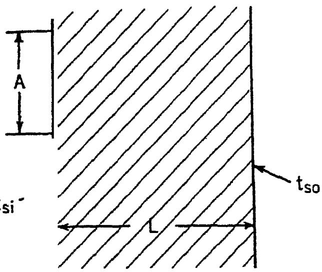

FIG. 5.—Wall consisting of one material only.

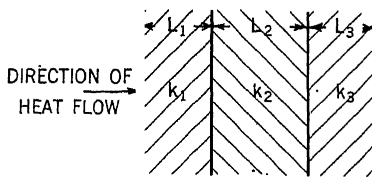

FIG. 6.—Wall with several layers in series.

(Fig. 6) is composed of several different materials (*e. g.*, a wall of various layers) the heat flow may be determined by:

$$Q_L = (t_{si} - t_{so}) \frac{1}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} +} \cdot A \tag{2}$$

where k_1, k_2, and k_3 are the conductivities of the various layers and L_1, L_2, and L_3 are their respective thicknesses. If the wall consists of several parallel heat paths (Fig. 7), then the total heat flow Q_L is given by:

$$Q_L = (t_{si} - t_{so}) \frac{1}{L} (A_1k_1 + A_2k_2 + A_3k_3 + \cdots) \tag{3}$$

This equation is an approximation only. It is based on the assumption that the various parallel heat paths through materials A_1, A_2, etc. do not influence each other. Actually, the temperature drop in each of these materials is different from that in other materials. Hence, at any given distance from the surface (say at level $C \cdots C$) different temperatures exist in the different materials, 1, 2, 3, etc., and lateral heat flow develops resulting in a larger value for Q_L than that obtained from Equation (3). An example of such a pattern has been described elsewhere by the author.[7] In furnace work, a very important case of parallel heat paths

[7] V. Paschkis, *Refrig. Eng.*, 47, 469 (1944).

is that of a metal with very high thermal conductivity running through the insulation. Such thermal short circuits are discussed on page 51.

If the two surfaces through which heat flows are not parallel and infinite, Equations (1) to (3) are no longer applicable, and other more

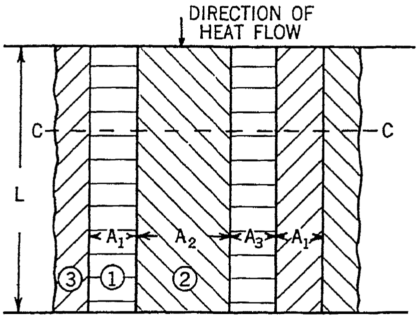

FIG. 7.—Several heat paths in parallel (composite wall; layers in parallel).

complicated forms must be used. Because of its wide applicability only one of these cases will be discussed here: heat flow through the wall of a cylinder. Assume (Fig. 8a) a pipe of length L, the length being so long, that the influence of the ends may be neglected. The inside diameter is d_i, the outside diameter d_o. The radial heat flow, Q_L, may be determined from:

$$Q_L = \frac{(t_{si} - t_{so})\pi L}{\frac{1}{2k}(\ln d_o - \ln d_i)} \tag{4}$$

If there should be several layers of insulation (Fig. 8b) the heat flow may be determined from:

$$Q_L = \frac{(t_{si} - t_{so})\pi L}{\frac{1}{2k_1}\ln\frac{d_1}{d_i} + \frac{1}{2k_2}\ln\frac{d_2}{d_1} + \frac{1}{2k_3}\ln\frac{d_o}{d_2} +} \tag{5}$$

More often than not the temperatures of the surroundings rather than the surface temperatures will be given. This occurs most frequently in the calculation of wall losses and will be dealt with later (page 36).

CONVECTION

If a liquid (or gas) is in contact with surfaces of different temperatures or if the body of liquid (or gas) has different temperature levels in different parts, heat flow will take place along with a movement of gas particles. The movement of the mass particles may be caused by either or both of the following: (1) the higher the temperature of the gas or liquid, the smaller its density, the difference in density between the

various parts causing a flow of particles, the heavier ones dropping, the lighter ones rising; (2) mechanical means, such as fans, pumps, etc., which may force the movement.

Relatively little is known about the laws which govern convection in furnaces. In general it can be said that transfer by convection increases with rising temperature difference, greater velocity, higher mean temperature, and greater fluidity of the liquid or gas (lower viscosity).

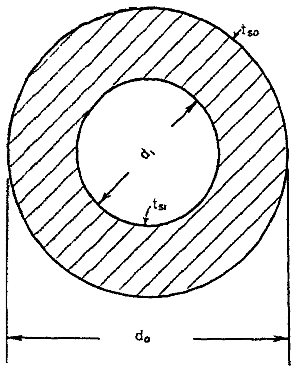

FIG. 8a.—Cross section through pipe insulation, single-layer wall.

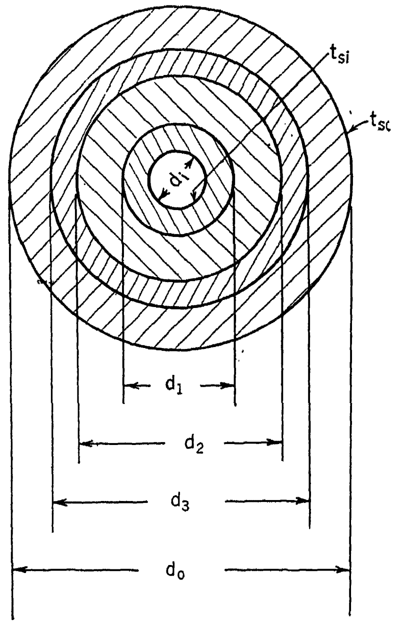

FIG. 8b.—Cross section through pipe insulation, multilayer wall.

Because of the uncertainties surrounding the mathematical expressions thus far developed for convection in furnaces, no formulas will be given here. (For further information, see books on heat transfer.[8] Some specific information is in Volume II of this book in connection with convection-type resistance furnaces.) Heat transfer by convection is influenced by the geometrical shape of the stream of gas or liquid, by the smoothness of the surfaces surrounding the stream and by the direction of the stream (vertical, horizontal, etc.)

RADIATION

Heat exchange by radiation depends on the following: (1) the temperature level at which the radiation takes place; (2) the temperature

[8] W. H. McAdams, *Heat Transmission*, McGraw-Hill, New York, 1942. A. Schack, *Industrial Heat Transfer*, Wiley, New York, 1933.

differences between the surfaces exchanging heat; (3) a physical constant, known as the "emissivity" of each surface; and (4) the geometry of the arrangement (shape and relative location of the surfaces). The influence of the temperature level and of the temperature difference can be

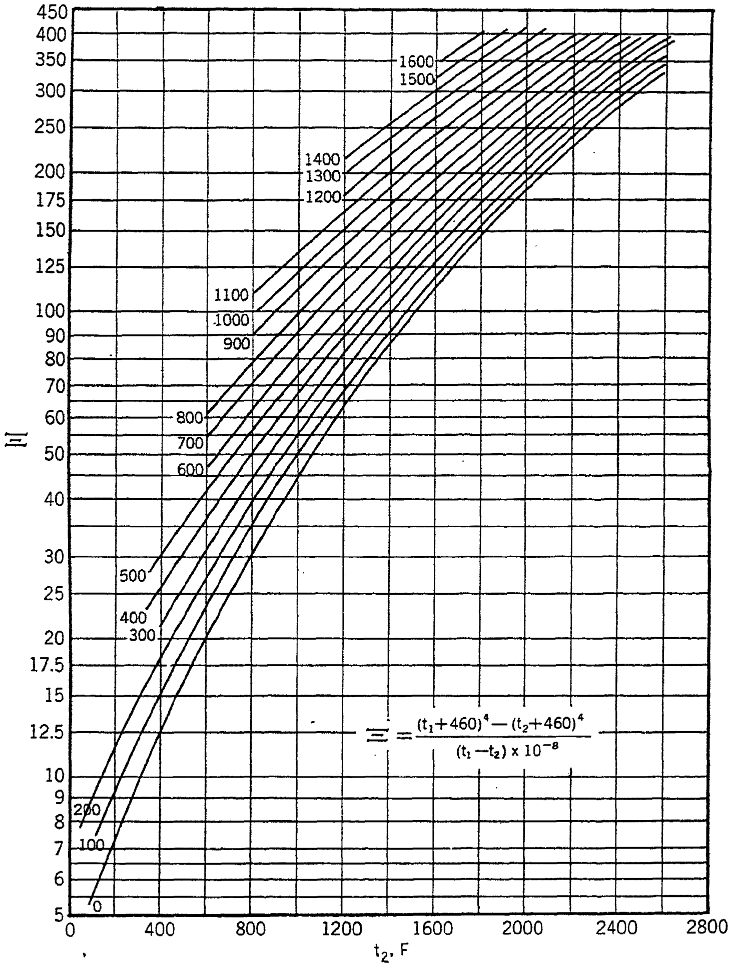

FIG. 9a.—Factor Ξ *vs.* temperature for temperatures up to 1800 F.

considered together. Heat flow due to radiation is proportional to the difference of the fourth power of the absolute temperatures of both surfaces.

If F designates the temperature of the surface in degrees Fahrenheit, its absolute temperature in degrees Rankine is given by:

$$R = 460 + F$$

For instance, if a body having a surface temperature of 1800 F radiates heat towards a body having a surface temperature of 400 F, the heat flow due to radiation is proportional to:

$$(460 + 1800)^4 - (460 + 400)^4 = 2.547 \times 10^{12}$$

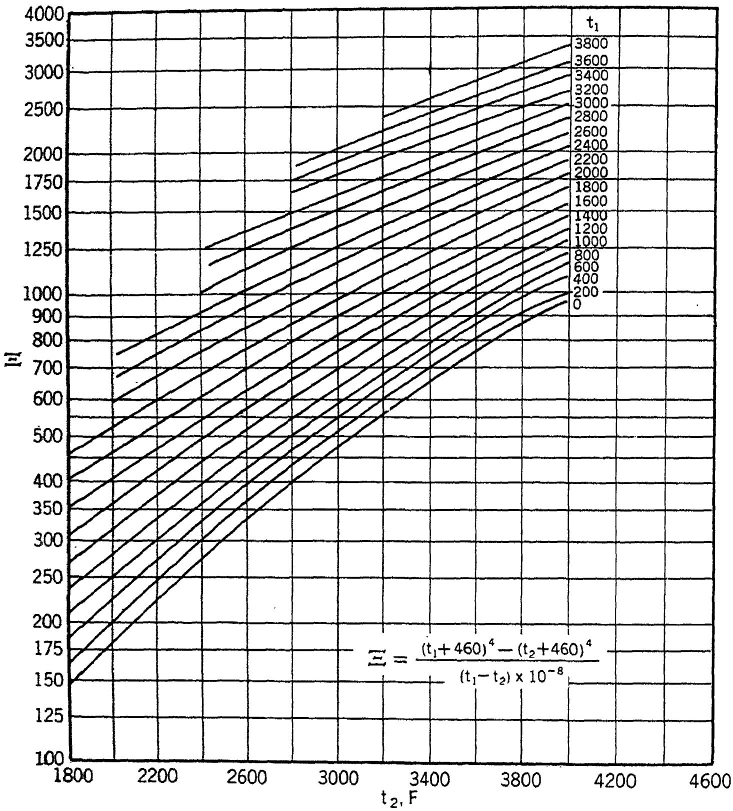

FIG. 9b.—Factor Ξ *vs.* temperature for temperatures above 1800 F.

It is somewhat inconvenient to use absolute temperatures and determine the fourth power for each case. It is therefore customary to say that heat flow due to radiation is proportional to the difference in temperatures in degree Fahrenheit times a factor, Ξ, which is in itself dependent on temperatures and which may be read from Figure 9. If t_1 and t_2 express the temperatures in degrees Fahrenheit of the surfaces involved in heat exchange, then:

$$\Xi = \frac{(460 + t_1)^4 - (460 + t_2)^4}{(t_1 - t_2) \times 10^8} \qquad (6)$$

Figure 9 is plotted in two parts (for different ranges of t_2) along the abscissa and contains curves for different values of t_1 in steps of 100 F (Fig. 9a) and 200 F (Fig. 9b).

The emissivity, ϵ, of a surface is a physical property indicative of the amount of radiation. It is the ratio of the emissive power of an actual surface to that of a "black body." The emissivity of the black body is called the Stefan-Boltzmann constant, σ. The emissivity of no surface can be larger than that of a black body, the physical expression of a body absorbing *all* the radiation received by its surface. In the English system, the Stefan-Boltzmann constant is 0.173×10^{-8} Btu per hr, sq ft, R^4. Values of the emissivities for various surfaces are tabulated in the appendix.

Concerning the geometry of radiation, only four cases will be considered:

(1) Two plane parallel surfaces of infinite areas having temperatures t_1 and t_2, respectively, exchange heat according to:

$$Q_{RAD} = (t_1 - t_2) \cdot \Xi \cdot \epsilon \cdot \sigma \tag{7}$$

(2) One body having the emissivity, ϵ_1, temperature t_1, and area A_1 entirely encloses a second body having emissivity ϵ_2, temperature t_2, and

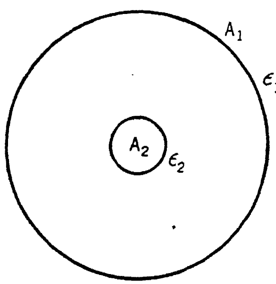

FIG. 10.—Heat exchange by radiation (two bodies, one entirely enclosed by the other).

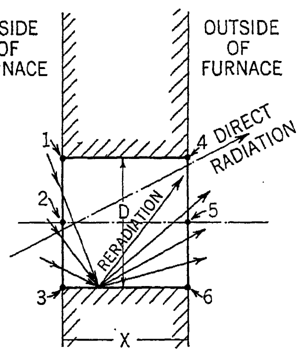

FIG. 11.—Geometric arrangement of an opening in a wall.[9]

area A_2 (Fig. 10). If ϵ_B is the emissivity of a black body, then the heat exchanged by radiation is given by:

$$Q_{RAD} = \frac{\sigma}{\dfrac{1}{\epsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\epsilon_2} - \dfrac{1}{\epsilon_B}\right)} \cdot \Xi \cdot (t_1 - t_2) \cdot A_1 \tag{8}$$

(3) An important case is the radiation through an opening, as through a door. Because of the reradiation at the sides, only a small amount of heat is lost, obtained by applying Equation (8) (with $\epsilon = 1$).

Hottel and Keller[9] have investigated this case. Figure 11 shows the general arrangement they studied. Figure 12 shows the total radiation factor as determined by them. In order to find the actual loss through an opening of a door:

$$Q_{door} = (t_1 - t_2)\cdot\Xi\cdot\tau\cdot\gamma\cdot A \tag{9}$$

Here τ is the total radiation factor from Figure 12, A the area of the door opening, and γ a factor allowing for additional losses by convection. The authors estimate γ between 1.04 and 1.09.

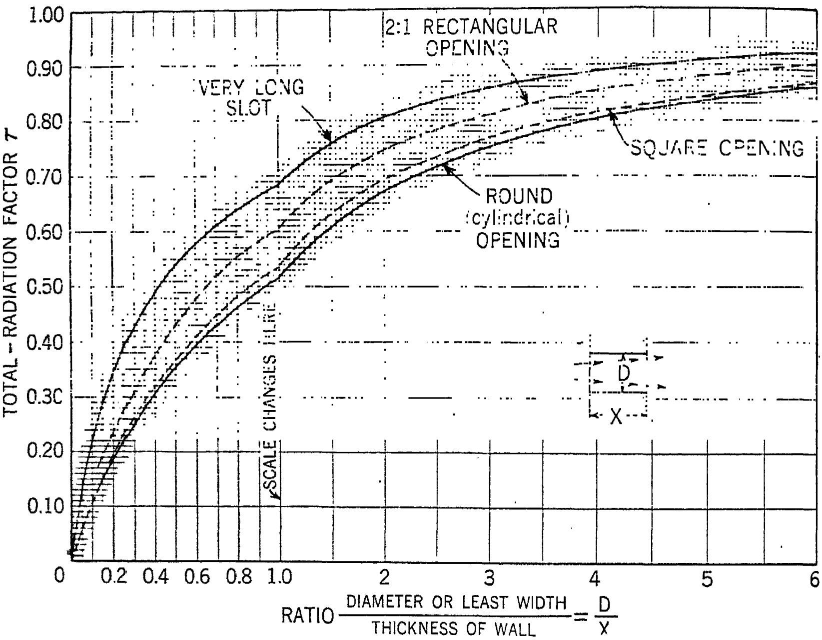

FIG. 12.—Factors for total radiation through openings of various shapes.[9]

(4) Finally, assume an arrangement as shown in Figure 13, in which two nonparallel elements of bodies at a distance r are exchanging heat by radiation. It is assumed that the areas of the two surfaces dA_1 and dA_2 are taken small enough to allow the use of average angles, φ_1 and φ_2. Then the heat exchange between the two is given by:

$$dQ_{RAD} = \frac{1}{\pi}\cdot\sigma\cdot\epsilon\cdot\Xi\cdot(t_1 - t_2)\cdot\frac{\cos\varphi_1\cdot\cos\varphi_2}{r^2}\cdot dA_1\cdot dA_2 \tag{10}$$

If the condition as to the small size of areas, dA_1 and dA_2, is not fulfilled, each corner (a and b, Fig. 13) will have a different angle, φ_1 or φ_2, and a different value of r.

[9] H. C. Hottel and J. D. Keller, *Trans. Am. Soc. Mech. Engrs., Iron and Steel*, 55, 39 (1933).

In most cases occurring in furnace work the geometry of radiation is rather involved. Therefore an integration of Equation (10) is, for engineering calculation, impracticable. Other methods of approach must be used. One employs models involving light radiation instead of heat radiation.[10] The bodies between which heat radiation is taking place are reproduced to scale. The amount of light radiation exchanged between them is measured, and by convenient "scale factors" the heat radiation can be determined. The method has been developed so far only under the assumption that the bodies do not conduct any heat. Under the influence of radiation, different parts of the bodies reach different temperatures, which will tend to equalize by conduction. The tendency to equalize is neglected in this method.

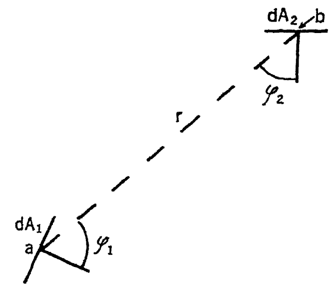

FIG. 13.—Cosine law for radiation between two elements.

A second method of approach makes use of an electrical analogy.[11] The bodies between which radiation occurs are considered as cut into elements and the radiation between all pairs of elements calculated by Equation (10). The various elements are then represented by a network of resistors. An example of this method is discussed in Volume II.

(b) *The Transient State*

It has been stated above that practical furnace problems almost always involve transient heat flow. This will now be illustrated by the following two examples.

(1) Consider a wall of a furnace which is cold at time "zero" (Fig. 14). The temperature of every point of the furnace wall is at the same value, t_a. Let the temperature on the inside be suddenly raised to temperature t_{si}. The temperature on the outside surface will not rise immediately. By plotting temperatures as ordinates against the wall thickness as abscissa for various times, a set of curves as shown in Figure 14 is obtained. Each curve shows the temperature distribution in the wall for a given time element. At time zero, the distribution is given by a horizontal line; for time "infinity," the distribution is shown by a straight inclined line.

Figure 15 shows the time–temperature curve for the outside surface. At time zero, when the inside temperature was suddenly raised from

[10] F. England and A. H. Craft, *Trans. Am. Soc. Mech. Engrs.*, 64, 691 (1942). M. Jakob and A. G. A. Hawkins, *J. Applied Phys.*, 13, 246 (1942).

[11] V. Paschkis, *Elektrotech. Maschinenbau*, 54, 617 (1936).

t_a to t_{si}, the temperature on the outside did not change immediately. It took a finite length of time for the "heat wave" to penetrate to the outside. Figure 16 illustrates the heat flow rate plotted *vs.* time. The upper curve shows the heat flow entering the inside surface; it starts with a high value and gradually drops to a constant, very much lower, value. The lower curve shows the rate of heat flow from the outside surface

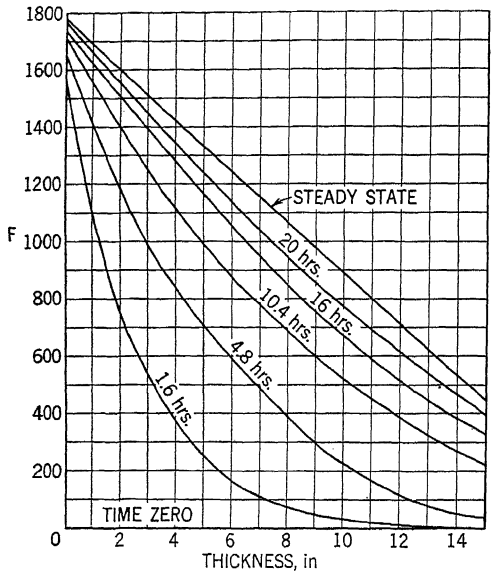

FIG. 14.—Temperature rise in a wall. Firebrick 15 in thick; t_{si}, 1800; t_o, 32; k, 0.833 Btu per ft, hr, F; c, 0.25 Btu per lb, F; ρ, 100 lb per cu ft. Each curve shows the temperature distribution in the wall at the time noted on the curve (time zero at the moment of starting temperature rise). The temperatures are taken above the cold ambient (32 F) as base.

to the surroundings; it starts from zero and gradually builds up to the same constant value. When both curves reach the same value, a steady flow of heat passes through the body. The same amount entering the wall at the inside surface leaves at the outside surface. A "steady state" has been reached. Up to that time a transient or "unsteady" state prevailed.

The example of the furnace wall is typical of one form of heat equalization taking place in furnaces. It is characterized by a constant temperature throughout the body (wall) at the start and a constant temperature gradient at the end.

(2) There is a second type of equalization of temperature. Assume a furnace initially at a constant elevated temperature. A ball is introduced into the furnace, the mass of the ball being small as compared with

the mass of the furnace walls. Neglecting the small temperature drop in the furnace, which is due to introducing the ball, the case will be treated as if the ball had been introduced into a space of constant temperature. The ball is to be held in midair in the center of the furnace and is therefore assumed to be heated uniformly from all sides. Its surface will receive heat by radiation or convection or both. The temperature at the center of the ball will not rise immediately. It starts slowly, accelerates, and then slows down again as the temperature of the

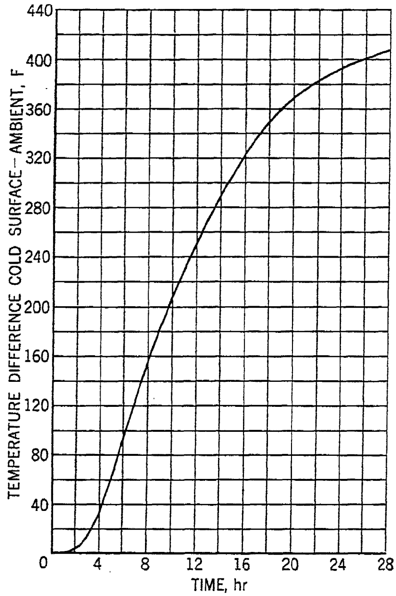

FIG. 15.—Time-temperature curve for the outside surface. (Wall of Figure 14.)

center approaches very slowly, and after a very long time, the temperature of the outside surface (Fig. 17). Uniformity is reached only after *infinitely* long time. The example of the ball is typical of the second form of heat equalization taking place in furnaces. It is characterized by a constant temperature throughout the body (ball) at the start and a constant but different temperature throughout the body (ball) at the end.

From these two examples it is obvious that, in furnace work, one always deals with the transient state.

The calculation of transient heat flow by *conduction* is extremely involved and has been developed for only very simple cases, so simple that they rarely occur in furnace work. Transient heat flow in liquids and gases based on *convection* has so far not yielded to mathematical treatment. *Radiation* by itself cannot be thought of as transient heat

flow because radiation takes place between surfaces alone, and the transient state is caused by mass effect, which is found only in bodies. Radiation and conduction can be combined in transient phenomena; unfortunately mathematical treatment is not yet available.

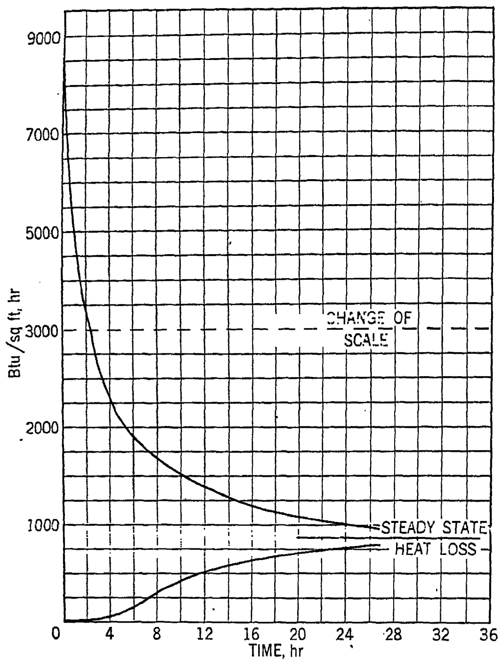

FIG. 16.—Heat flow for wall as in Figure 14 (inside surface raised suddenly to 1800 F, ambient 32 F). Upper curve, rate of heat flow entering inner surface; lower curve, rate of heat loss from outer surface.

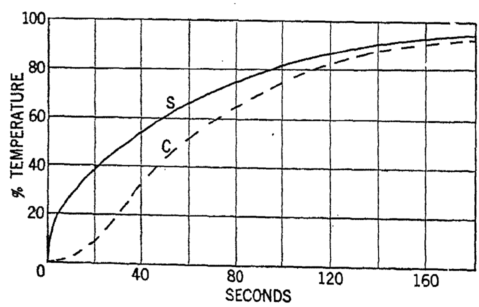

FIG. 17.—Temperature rise in center and at surface of steel ball (temperatures are indicated in per cent of furnace temperature). S, surface; C, center. Physical characteristics: thermal conductivity, 30 Btu per ft, hr, F; density, 485 lb per cu ft; specific heat, 0.150 Btu per lb, F; boundary conductance, 200 Btu per sq ft, hr, F.

Three methods of analysis are known (besides the mathematical analytical method, which is available only for certain simple cases):

(1) A graphical method, first devised by Schmidt.[12] It is suitable only for very simple cases and only after making a number of simplifying assumptions. Even then, its use is rather cumbersome.

(2) A numerical method, known as Southwell's relaxation method and applied to heat flow problems by Emmons.[13] The method is easy to learn and—within its range of applicability—is very useful. It becomes quite involved in case of changing thermal properties and if applied to cyclic heating.

(3) An electric analogy method, which is very versatile and subject to fewer limitations than the other methods. Its main drawback is its rather expensive equipment (available at *Department of Mechanical Engineering, Columbia University*, New York City). The basic equations between transient heat flow in solids and certain electric circuits are identical. Hence, by setting up certain electric circuits and carrying out electric measurements, heat problems may be analyzed which cannot be solved mathematically. The method, first devised by Beuken,[14] has been widened in its scope and introduced into the United States by the author.[15]

2. The Primary Heat Problem of Electric Furnace Design

Every electric furnace is subject to a very typical heat flow problem: heat is generated in one well-defined zone and performs two functions, heating the load and covering the heat losses. Exception must be made for furnaces in which the load is introduced at its highest temperature and is either held constant for some time at this temperature or is allowed to cool at a predetermined rate. The heat flow for losses is rather obvious and may in many cases be considered as a steady-state flow. The flow towards the charge is always a transient phenomenon. This fact, too often overlooked, cannot be stressed enough.

Figure 18 is a series of diagrammatic sketches of various furnace types. In each sketch the direction of heat flow *in* the load is shown by the density of shading, the parts of highest temperature being darker, the parts of lower temperature being lighter. Heat flow is naturally directed from the high to the low temperatures.

The arrows show the direction of flow of the heat which is lost. Figure 18*a* shows an *arc furnace;* note the temperature drop between the electrodes and that the temperature drop is from the electrode toward the wall. Figure 18*b* shows *coreless induction furnace,* in which heat is generated in a very thin layer only and is transmitted toward the inside of the load. Figure 18*c* is an *induction furnace with core,* heat being

[12] E. Schmidt, *Foeppl's Festschrift,* Springer, Berlin, 1932, p. 179. See also W. H. McAdams, *Heat Transmission,* McGraw-Hill, New York, 1942, p. 39.

[13] H. W. Emmons, *Trans. Am. Soc. Mech. Engrs.,* 65, 607 (1943).

[14] C. L. Beuken, *Economisch Technisch Tijdschrift,* 17 (Jan., 1937).

[15] V. Paschkis and H. D. Baker, *Trans. Am. Soc. Mech. Engrs.,* 64, 105 (1942).

generated in the channel and moving by convection toward the batch at the top of the furnace. Figure 18*d* is an *indirect resistance furnace.* Heat is generated in the resistors and is transmitted by radiation to the outside of the load, from whence it is transmitted mostly by conduction to the interior. Special types of resistance furnaces, *e. g.*, those having forced air circulation, salt baths, etc. will be discussed later. Figure 18*d* also applies to continuous furnaces, in which the load moves through the

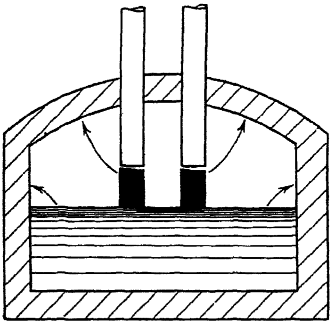

FIG. 18a.—Indirect-arc, direct resistor furnace.

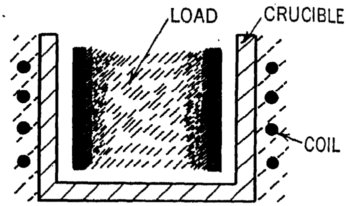

FIG. 18b.—Coreless induction furnace.

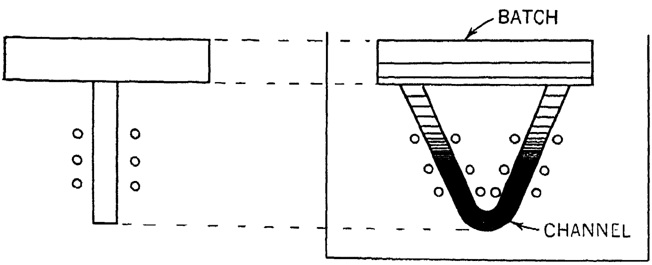

FIG. 18c.—Induction furnace with core.

furnace. The indicated heat flow occurs in each cross section of the furnace through which the load passes. The heat flow in each section takes place on a different temperature level, the differences becoming smaller as the load progresses towards the furnace exit. Figure 18*e* shows a *direct-heat resistance furnace.* Even there the load is subject to heat flow, which is, however, perpendicular to the main direction of electric flow and serves to cover the heat losses. A cross section is shown, therefore, rather than a longitudinal section. Figure 18*f* is an *HFC furnace.* Although heat is generated uniformly throughout the load, there is a heat flow to the surface because the surface loses heat to the surroundings.

In the following considerations, the heat which is used to cover the losses is assumed to be constant, that is, the investigations are limited to

continuously operated furnaces. The inevitable temperature drop in the furnace when being loaded is not considered: the furnace temperature is assumed to be kept constant. This assumption allows concentration on the heat flow *in* the load. In Figure 17 the core temperature did not quite reach the surface temperature at the end of the curve. In fact, it takes infinite time to bring the core of any body which is heated from the surface to the same temperature as the surface. Within a finite time,

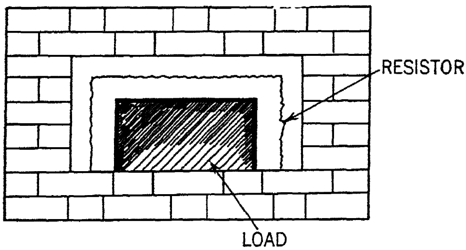

Fig. 18d.—Indirect-heat resistor furnace.

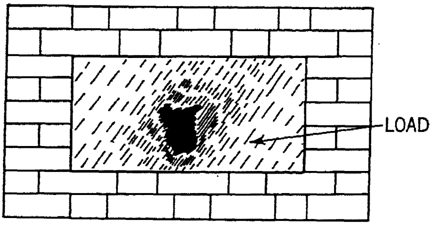

Fig. 18e.—Direct-heat resistance furnace.

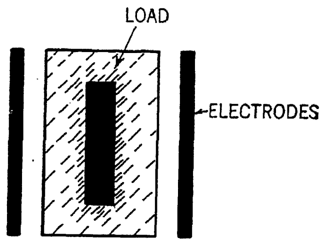

Fig. 18f.—High-frequency capacitance heating.

the difference between core and surface temperatures becomes very small, fractions of a degree, but nevertheless still exist. This is of paramount importance. From this fact it becomes apparent that any specification calling for one temperature of the load is incomplete. In order to use finite heating times, a compromise must be made: temperature differences in the load must be allowed. A complete specification must indicate the permissible temperature differences in the load at the end of the heating period. If the furnace temperature is not kept constant, another variable is introduced: the same temperature difference in the load at the end of the process can be achieved in different times. However, a shorter time means a steeper temperature gradient, which implies higher rate of heating. Therefore the specifications should also include permissible rates of heating.

Metallurgical and ceramic science is not yet far enough advanced to be able to give the permissible temperature differences and rates. Some

empirical rates are found in industry but are too vague, however, and are based on rule-of-thumb. The correct procedure would be determining the metallurgically and ceramically desirable rates and then finding out if and to what extent the laws of heat transmission permit the application of these desirable rates for given sizes and shapes. Recognition of these facts will help avoid errors due to the influence of the size of the pieces which make up the load. The problem takes on a different aspect for each furnace type and will be discussed later at greater length. At present, it is sufficient to remember that the larger the load the longer is the time necessary to obtain a desired degree of uniformity: hence the larger the furnace, the longer the time required to bring the charge in the furnace to a given degree of uniformity.

This leads to the fundamental problem of furnace design: Given a desired output (in pieces or tons per unit of time), given the material involved, given the maximum permissible temperature difference at the end of the heating time, given the maximum permissible rate of heating, what is the type and size of the furnace having the smallest over-all operating cost? This problem is sometimes hidden and sometimes formulated clearly; but in every case it is advisable to work it out as far as possible. From all the types of furnaces ("types" refers here not only to arc, induction, and resistor furnaces, but also to the various forms of each of these types) there is one size best suited to the conditions which gives minimum heat consumption. This furnace need not necessarily be the most economical; the question of production reserve, of space, of labor, etc. can result in lower over-all costs for a furnace even if this furnace has higher power consumption. Yet it is important to find the furnace of lowest heat consumption.

This analysis can be conveniently made by means of the basic diagram in Figure 19. Plot on the abscissa the capacity of the furnace chamber. Over this common abscissa a number of curves are to be drawn, the first showing the heating time necessary to keep the temperature difference in the charge and the permissible heating rate within the specified limits (Curve 1). If the charge is to be held at constant temperature after having reached the given temperature uniformity (*e. g.*, for the purpose of refining in a melting furnace or for the purpose of carburization in a case-hardening furnace), draw a new curve (1*a*) including the "holding time." This curve is parallel to Curve 1, only moved upward in every point for the constant value of the holding time. Any value of the abscissa divided by the corresponding value of Curve 1*a* gives the output of one furnace. If the abscissa is in cu ft and Curve 1 and 1*a* in hr, then the output will be in cu ft per hr. Divide the total desired output by the output per furnace to find the number of furnaces necessary to obtain the total output. This determination must be made

for various values of the abscissa. Thus, point by point, a curve is obtained for "number of furnaces *vs.* furnace capacity" (Curve 2). Plotting the heat losses per hour for each furnace capacity yields Curve 3. By multiplying the rate of heat loss (Curve 3) by the total heating time (Curve 1*a*), the heat losses of each furnace during the heating

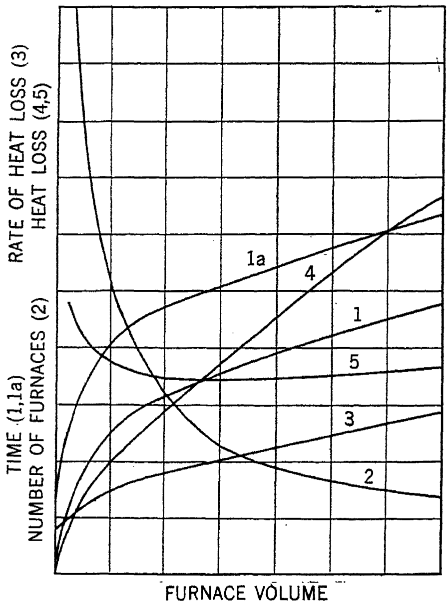

Fig. 19.—Basic diagram for determining the furnace size of smallest heat cost for a given output. Curve 1, heating time; 1*a*, heating plus holding time; 2, number of furnaces; 3, rate of heat loss per furnace; 4, total heat loss per furnace; 5, total heat loss for entire output.

time are obtained (Curve 4). Finally, by multiplying (for each value of the abscissa) the ordinates of Curve 4 (total heat losses for one furnace) with those of Curve 2 (number of furnaces), the ordinates for Curve 5, showing the total heat losses for all furnaces, are obtained. The minimum will show the best furnace capacity from the viewpoint of heat consumption.

These considerations call for relatively complicated calculations, which will frequently not be carried out in the design of furnaces. It is advisable however—even if the curves cannot actually be determined—always to have these relations in mind. The determination of Curves 1 and 1*a* will be different for the various furnace types and will be considered individually later. Curve 2 follows directly from Curve 1. However the calculation of heat losses (Curve 3) has so much in common

for the various furnace types that it will be discussed here in general, leaving only some special questions for the individual furnaces.

3. Heat Losses

(a) *Steady-State Losses*

First the heat losses through *infinite plates and cylinders* of infinite length shall be considered, then the influence of the shape of the furnace chamber (shape factor).

HEAT LOSSES IN WALLS

As mentioned above, the thermal resistance of a wall is composed of the resistance of the wall material proper (as expressed by Eqs. 1 to 3) and the boundary resistance (film resistance). On the hot side of the wall, the temperature of the surroundings is given more often than the temperature of the wall. Exception must be made for melting furnaces, in which walls are in direct contact with liquid metal. In all other cases, the temperature, t_i, of some part other than the wall is given (*e. g.*, of resistors, gases, etc.). From this part, the heat is transmitted to the inside surface of the wall, which in turn conducts the heat towards the outside. In heat-treating furnaces, the calculation of heat losses is based on the temperature of the resistors. In the roofs of arc furnaces the temperature of the bath, and in part the temperature of the arc, serve as basis for the calculation of the heat flow. This thermal flow meets a resistance on its way from the source of heat to the inside surface (inside film resistance). This resistance is a function of temperature.

Similar conditions prevail on the outside of the wall. The temperature, t_o, of the surrounding air is given. The outside surface of the wall is at a higher temperature than that of the surroundings. Hence a heat flow occurs which is subject to a resistance (outside film resistance). This resistance is a function of the temperature difference and the absolute level of temperature. Heat transfer to the inside surface occurs by radiation and convection, that from the outside surface mostly by convection only, a smaller part being transmitted by radiation.

Denote the resistance of the "film" on the inside by $1/h_i$ and that on the outside by $1/h_o$. Then the total resistance to the flow of heat through a wall composed of several layers can be expressed by:

$$R_h = \frac{1}{A}\left(\frac{1}{h_i} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_o}\right) \tag{11}$$

using the same notations as in Equations (1) and (2). The heat losses are found from:

$$Q_L = \frac{t_i - t_o}{R_h} \tag{12}$$

For a cylinder of great length the heat loss equation (page 36) becomes:

$$Q_L = \frac{(t_1 - t_2)\pi L}{\dfrac{1}{d_i h_i} + \dfrac{1}{2k_1}\ln\dfrac{d_1}{d_i} + \dfrac{1}{2k_2}\ln\dfrac{d_2}{d_1} + \dfrac{1}{2k_3}\ln\dfrac{d_3}{d_o} + \dfrac{1}{d_o h_o}} \qquad (13)$$

The influence of end effects is discussed together with the shape factor (page 44). Values for h_o as a function of temperature difference are given in Figure 20.

Heat loss calculations following Equations (1) to (5), neglecting the influence of the film conductance, are much simpler than calculations following Equations (11), (12), and (13), which take the inside film conductance into account.

In order to know when the simpler equations can be used, the error involved in this simplification must be determined. The error, p (in per cent), due to neglecting the film resistance can be expressed by:

$$p = \frac{100k}{L}\left(\frac{1}{h_o} + \frac{1}{h_i}\right) \qquad (14)$$

Conductivity values for different materials, as well as values of film conductance, are not known very accurately, probably not even within 10%. Therefore a simplified calculation is reasonable if the results are within this range of 10% accuracy (that is, $p = 10$). In practical furnace design, $\frac{1}{h_o}$ will seldom be higher than 0.5 and $\frac{1}{h_i}$ seldom higher than 0.12. With these figures, the equation yields a value of $k/L = 0.161$. If $k/L \leqq 0.161$, then the error due to neglecting the film conductances will be smaller than, or equal to, 10%.

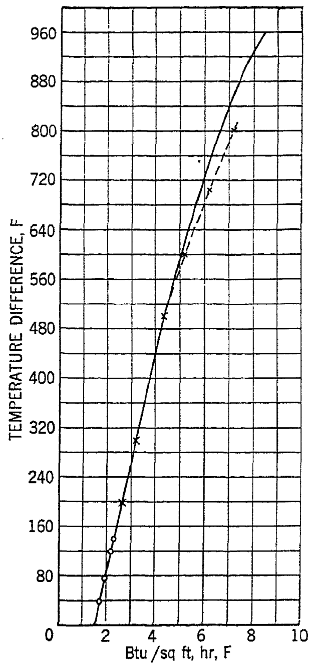

FIG. 20.—Heat-transfer coefficient (film conductance) *vs.* temperature difference between surface and ambient. Ambient at 70 F. The full line is based on figures given by the R. Bosch Co. The values check closely with those by R. N. Stansel (values indicated by circles) and those given by Trinks.[15a] Trinks' values at high temperatures (broken line) deviate slightly.

While it is frequently possible to neglect the film resistances in determining the steady-state heat losses, they cannot be neglected in deter-

[15a] R. N. Stansel, *Industrial Electric Heating*, Wiley, New York, 1933. W. Trinks, *Industrial Furnaces*, 3rd ed., Vol. I, Wiley, New York, 1934.

mining the temperature drop. The total temperature difference between inside and outside ($t_i - t_o$) is the sum of all the various (thermal resistances) However, the slope is not uniform in the various materials.

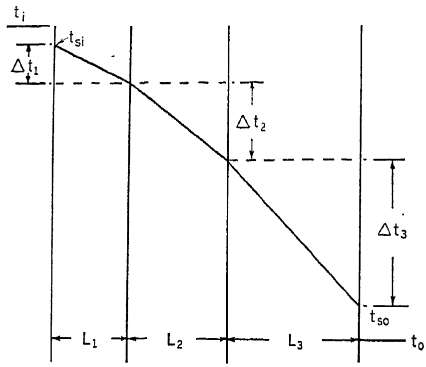

FIG. 21.—Nomenclature for temperature drop in a multilayer wall.

Referring to Figure 21, the temperature drop in the first layer can be found from:

$$\Delta t_1 = \frac{\frac{L_1}{k_1}}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \cdots \frac{1}{h_o} + \frac{1}{k_i}} \cdot (t_i - t_o) \tag{15}$$

For any other layer the drop is correspondingly

$$\Delta t_n = \frac{\frac{L_n}{k_n}}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \cdots \frac{1}{h_o} + \frac{1}{k_i}} \cdot (t_i - t_o) \tag{15a}$$

A knowledge of the temperature distribution in the wall is necessary for two reasons: to select the best suited materials; and to find the correct values for the conductivities involved. Materials withstanding higher temperatures have higher thermal conductivities and are therefore less efficient; a knowledge of the temperature distribution in the wall makes it possible to limit the use of the less efficient materials to those parts of the wall where they are needed because of their refractoriness. Since thermal conductivities change with temperature, a knowledge of the temperature distribution is essential in order to select the correct conductivity values.

Example.—A wall is to be designed for a furnace temperature of 2200 F and an ambient temperature of 70 F. The inside layer is to be made of fire brick, the outside layer of insulating block. The block will result in low heat losses; however it may not be exposed to temperatures higher than 1900 F. In order that the temperature at the interface between firebrick and block may not exceed 1900 F, the thickness of the firebrick must be so determined as to yield a temperature drop of at least 300 F. A higher drop is permissible from the viewpoint of the use of the block material; but a higher drop means a thicker layer of firebrick and consequently increased heat storage, increased losses in intermittent service, and, because of the shape factor, even increased

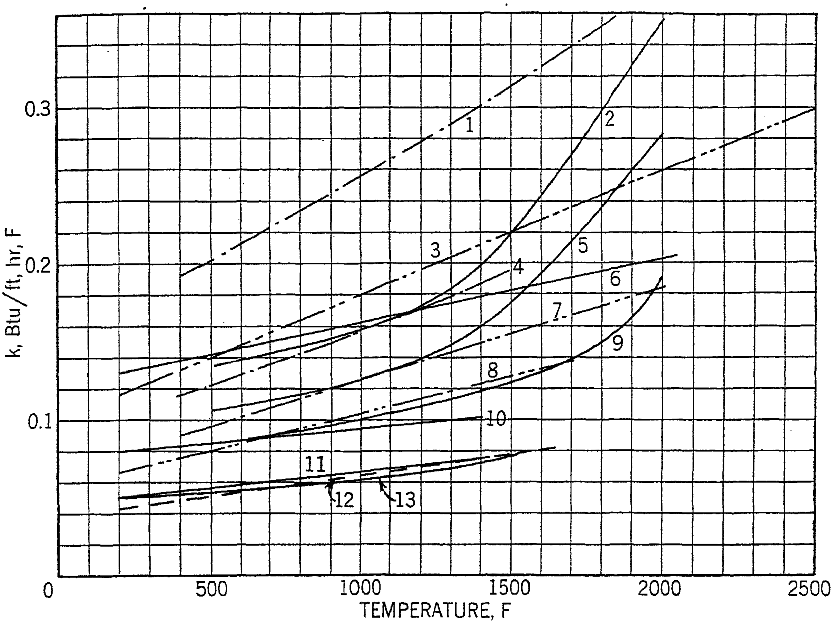

FIG. 22.—Thermal conductivities of various insulating materials: (1) B & W, K 30–3; (2) JM, 26; (3) Armstrong A, 25; (4) B & W, K 20; (5) JM, 23; (6) JM, superbrick; (7) Armstrong, A20; (8) Armstrong, A16; (9) JM, 20; (10) JM, natural brick parallel to strata; (11) JM, natural brick perpendicular to strata; (12) Cil-o-Cel, powder; (13) Block.

heat losses in the steady state. The temperature drop of 300 F includes also the small difference between inside furnace temperature (t_i) and inside surface temperature (t_{si}).

In order to determine the thickness of the firebrick, the conductivities and film conductances must be known. Figures 22 and 23 show the thermal conductivities of insulating material and refractory materials, respectively. (The determination of thermal conductivities at elevated temperatures is quite difficult, and following different methods of investigation gives different results. This should be kept in mind when comparing the different curves in Figures 22 and 23. The curves are those given by the manufacturers, who use different

methods of determination. See Nichols.[16] Recently a standard method of testing was adopted by the ASTM.[17]

Figure 20 shows the outside film conductance given as function of temperature. In order to select the proper conductivities and film resistances, the temperatures must be estimated, insofar as they are not given: inside temperature, 2200 F; inside surface temperature, estimated, 2150 F; firebrick–block interface, 1900 F; outside surface temperature, estimated, 200 F; and ambient temperature, 70 F. The inside film conductance is considered as "radiation only." It is determined using Equation (7) and Figure 9*b*. Black-body radia-

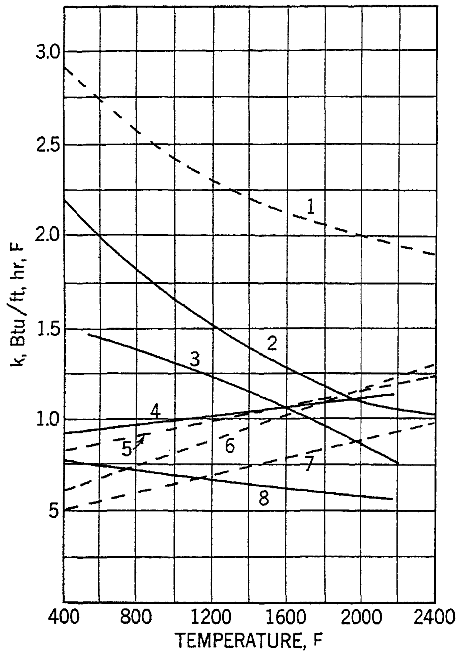

FIG. 23.—Thermal conductivities of various refractory and insulating materials: (1) H.-W.-R. Co. magnesite brick; (2) burned magnesite; (3) Ritex magnesite; (4) burned chrome brick; (5) H.-W.-R. Co. chrome brick; (6) Star silica brick; (7) Ritex chrome brick; (8) fireclay brick.

tion may be assumed ($\sigma = 0.173 \times 10^{-8}$). From Figure 9 for a temperature of 2200 F radiating to a temperature of 2150 F, it follows that $h_i = 0.173 \times 10^{-8} \times 740 \times 10^8 = 128$. From Figure 20 for a temperature difference of $200 - 70$ F = 130 F, it follows that $h_o = 2.27$. For a mean temperature of $\frac{1}{2}(1900 + 2150) = 2075$ F, the conductivity of firebrick is found from Figure 23 to be 0.895 Btu per ft, hr, F. Similarly, for insulating block at a mean temperature of $\frac{1}{2}(1900 + 200) = 1050$ F, the conductivity (Fig. 22) is 0.063 Btu per ft, hr, F. Substituting these values in Equation (~~14~~) and solving for

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[16] P. Nichols, *Bull. Am. Ceram. Soc.*, 15, 37 (1936).

[17] C177–42T and C182–43T.

These temperatures are sufficiently close to the values assumed above, upon which the conductivities introduced in the equation were based. If the agreement were not so close, the procedure would have to be repeated, using the conductivities for the latest calculated temperatures. The resulting temperatures would be closer to the correct values than the previous ones. In the final calculation, the temperatures should be so close to those in the preceding calculation that the conductivities are practically no longer influenced by the change in temperature. With a little experience, two or, at the most, three calculations should be sufficient.

It has been explained above (page 20) how to proceed in the case of several parallel heat paths. Equations (11) and (12) can be applied correspondingly. It should not be forgotten, however, that because of temperature differences in neighboring layers a thermal flow will occur perpendicular to the main flow (in other words, parallel to the furnace surface). This transverse heat flow always increases the total heat flow as compared with the heat flow resulting from two parallel streams without mutual interaction (see page 52).

SHAPE FACTOR

So far, walls and cylinders of infinite length have been considered. In practice, of course, only finite bodies occur, the most common one being the parallelepiped.

For any finite shape, the equation for steady-state heat loss can be written:

$$Q_L = (t_{si} - t_{so}) \cdot k \cdot S \tag{16}$$

where S represents the "shape factor" and has the dimension, "length." It contains the ratio area/thickness.

For walls composed of several layers, the heat losses may be determined from:

$$Q_L = (t_{si} - t_{so}) \cdot \frac{1}{\dfrac{1}{S_1k_1} + \dfrac{1}{S_2k_2} + \dfrac{1}{S_3k_3} + \cdots} \tag{17}$$

S_1, S_2, S_3, etc. representing the shape factors of the various layers, while k_1, k_2, and k_3 represent the respective conductivities. Since the exact shape factor is not known for heat flow through the walls of a parallelepiped, various shape factors have been proposed based on the assumption that surfaces parallel to the outside (or inside) are isotherms. Such shape factors are reviewed below. The final working formula adopted here for the shape factor of a parallelepiped is:

$$S = \frac{A_i}{L}\sqrt{1 + 12\frac{L}{\sqrt{A_i}} + 24\left(\frac{L}{\sqrt{A_i}}\right)^2} \tag{18}$$

where A_i is the area of the inside wall surface and L is the wall thickness. The derivation of this formula may be found below.

For walls composed of several layers, S may be found for each layer by introducing for A_i the inside surface, and for L, the thickness of the layer. If the root:

$$\sqrt{1 + 12\frac{L}{\sqrt{A_i}} + 24\left(\frac{L}{\sqrt{A_i}}\right)^2} = K$$

then $S = (A_i/L)K$. K is plotted as a function of A_i and L in Figure 24. Equation (18) is an approximation for most furnace sizes.

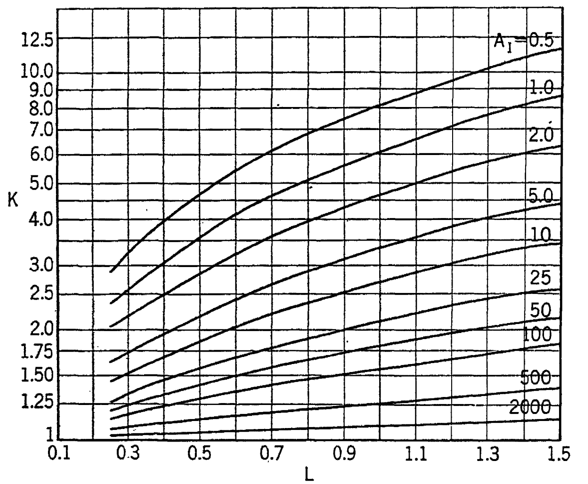


FIG. 24.—Factor K for rectangular furnaces of inside area A and wall thickness L.

$$K = \sqrt{1 + 12\frac{L}{\sqrt{A_i}} + 24\left(\frac{L}{\sqrt{A_i}}\right)^2}$$

Equation (18) cannot be applied to cylindrical or irregularly shaped furnaces. In this case the areas of the outside and inside surfaces (A_o and A_i) should be determined. The shape factor is then the geometric mean between the two areas, divided by the wall thickness of the layer:

$$S = \sqrt{A_i A_o}/L \tag{18a}$$

In a *rectangular* furnace of uniform insulation, the thickness of insulation is well defined. However there is a question as to which value should be introduced for the area in Equation (18). Jakob[18] suggested the following list of possibilities: arithmetic mean between inside and outside area; geometric mean between inside and outside area; area of surface in middle of wall thickness; volume of insulation divided by thickness; and an area defined by Langmuir.[19] Let A denote the area to be introduced into Equation (18), A_i the

[18] M. Jakob, *Z. ges. Kälte-Ind.*, **34**, 146 (1927).

[19] I. Langmuir, E. Q. Adams, and G. S. Meikle, *Trans. Am. Electrochem. Soc.*, **24**, 53 (1913).

inside area of the furnace, L the thickness of the wall, l the total length of the three sides $[l = 4\,(a + b + c)]$, f a function, defined by $l = f\sqrt{A_i}$, and S the shape factor. The expression $S = A/L$, using the various possibilities given above, can then be found from one of the following equations:

$$\frac{A}{L} = \frac{A_i}{L}\left(1 + \frac{Ll}{A_i} + \frac{12L^2}{A_i}\right) \tag{19}$$

$$\frac{A}{L} = \frac{A_i}{L}\sqrt{1 + \frac{2Ll}{A_i} + \frac{24L^2}{A_i}} \tag{20}$$

$$\frac{A}{L} = \frac{A_i}{L}\left(1 + \frac{Ll}{A_i} + \frac{6L^2}{A_i}\right) \tag{21}$$

$$\frac{A}{L} = \frac{A_i}{L}\left(1 + \frac{Ll}{A_i} + \frac{8L^2}{A_i}\right) \tag{22}$$

$$\frac{A}{L} = \frac{A_i}{L}\left(1 + 0.54\frac{Ll}{A_i} + \frac{1.2L^2}{A_i}\right) \tag{23}$$

Each of these equations contains three variables: A_i, L, and l. By introducing the function, f, defined by $f = l/\sqrt{A_i}$, where f is a function of b/a and c/a, the number of the variables can be reduced to two: $L/\sqrt{A_i}$ and f, because:

$$\frac{Ll}{A_i} \qquad \frac{Ll}{\sqrt{A_i}} = \frac{L}{\sqrt{A_i}}\cdot f \quad \text{and} \quad \frac{L^2}{A_i} = \left(\frac{L}{\sqrt{A_i}}\right)^2$$

A short review as well as a detailed analysis [20] shows that Equations (19), (21), and (22) agree closely. Equations (20) and (23) form a second group giving results which also differ very little. Between the two groups of formulas the differences are more appreciable, varying from 15 to 30% under normal insulating conditions ($L/\sqrt{A_i} = 0.2$ or smaller) and from 250 to 350% at $L/\sqrt{A_i} = 1.0$.

The author's experience shows that operating with Equation (20) yields results conforming closely to actual measurements. Therefore this formula is used here throughout.

By introducing f and $L/\sqrt{A_i}$, Equation (20) can be written as follows:

$$\frac{A}{L} = \frac{A_i}{L}\sqrt{1 + 2f\frac{L}{\sqrt{A_i}} + 24\left(\frac{L}{\sqrt{A_i}}\right)^2} \tag{20a}$$

Dealing now with this equation alone, the magnitude of f will be investigated. Stipulating that $b > c > a$, it follows that b/a and c/a are always greater than, or equal to, 1. A cube has the smallest value for f, namely $f = 4.9$. Figure 25 gives the values of f as a function of the ratios, b/a and c/a. Obviously it would be very convenient if f were to have a constant value. On page 43, in Equation (18), $f = 6$ = constant is used. The value of $f = 6$ has been selected deliberately. The choice was influenced by the desire to find a value covering as large a range of probable ratios, a/c and b/c, as possible without

[20] V. Paschkis, *Elektrowärme*, **3**, 157, 251 (1933).

incurring too great an error. Figure 26 illustrates this error. $L/\sqrt{A_i}$ is plotted as abscissa and a correction factor (error) for various values of f as ordinate. This correction factor is the ratio of the values of the root in Equation (20a) for $f \neq 6$ to the value of the root for $f = 6$. For all ordinary insu-

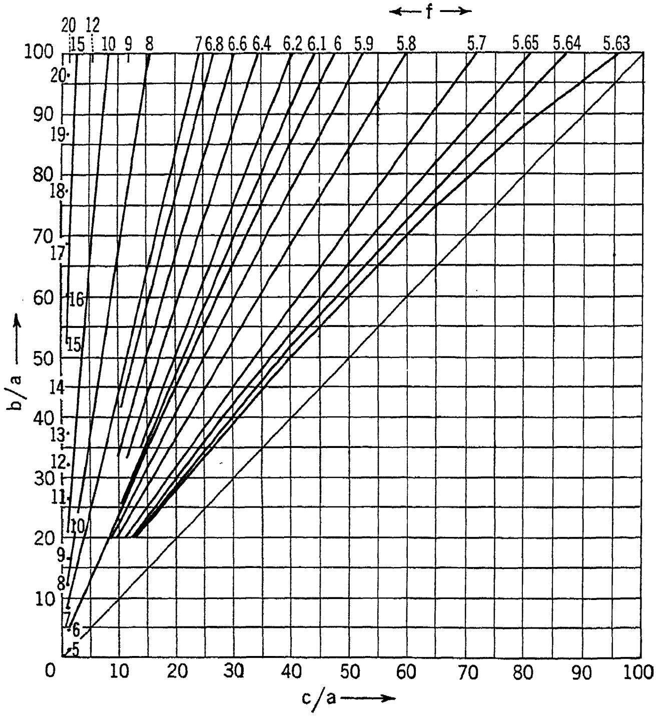


FIG. 25.—Factor f (parameter) plotted *vs.* c/a and b/a. Select $b > c > a$ so that $b/a > c/a > 1$; a, b, c, sides of furnace (inside).

lating conditions, the error will be less than 10%. Moreover, for greater accuracy the error as read from Figure 26 can be used to correct the values in Equation (18).

The Selection of Furnace Shape.—The wall losses naturally depend on the shape of the furnace as well as on its size. The influence of the shape is so great that it is often more advantageous to select a larger furnace of the "economical" shape than a smaller furnace of a less favorable shape. The most important and most striking example is that of rectangular and cylindrical furnaces, although (mainly for melting purposes) odd shapes also, *e. g.*, furnaces with elliptical bases, may be considered.

Returning to the comparison between rectangular and cylindrical furnaces, the latter, even if larger in inside volume, is often more advan-

tageous. Two influences contribute to the same result: first, for a given inside volume the cylindrical furnace has a smaller inside surface; and

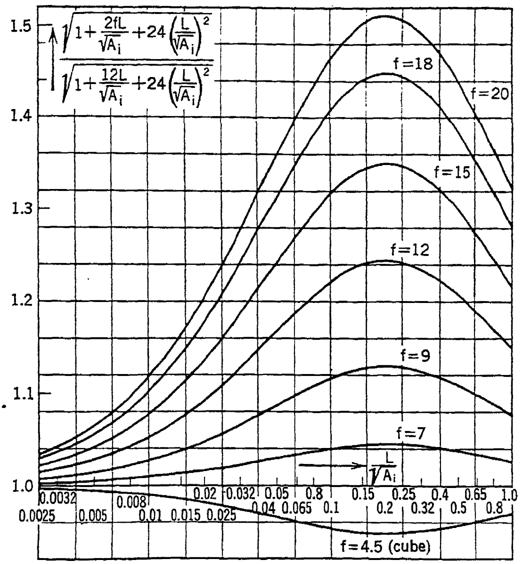


FIG. 26.—Form factor K for $f = 6$ and for f = variable. Ordinates show ratios of K for $f = 6$ to K for f = variable.

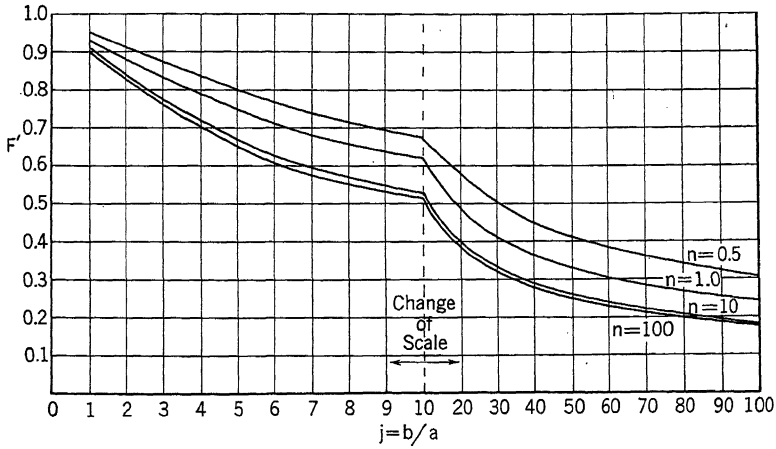


FIG. 27.—Ratio of inside areas for cylindrical and rectangular furnaces of equal volume.

$$F' = \frac{\text{area of cylindrical furnace}}{\text{area of rectangular furnace}}$$

$$n = h/d_i \qquad j = b/a$$

where h is the length of the cylinder, d_i its inside diameter, and b and a the sides of the rectangular furnace.

second, for a given inside surface area and a given wall thickness the cylindrical shape frequently yields lower steady-state wall losses than the rectangular furnace. It must be noted however, that only furnaces of the same length should be compared. Figure 27 shows curves comparing the inside area of a cylindrical furnace with that of a rectangular furnace, both having the same length and the same inside volume. In order to

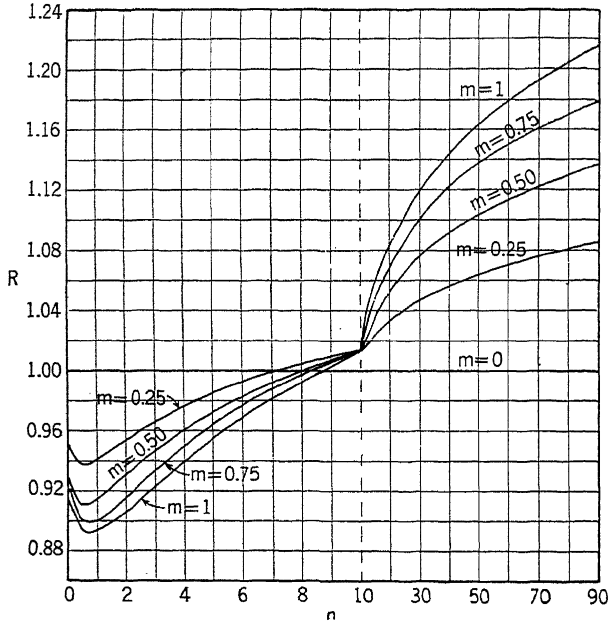


FIG. 28.—Ratio of heat losses of cylindrical and rectangular furnaces.

$$R = \frac{\text{losses of cylindrical furnaces}}{\text{losses of rectangular furnaces}}$$

$$m = 2L/d_i \qquad n = h/d_i$$

where h is the length of the cylinder, L the thickness of the insulation, and d_i the inside diameter.

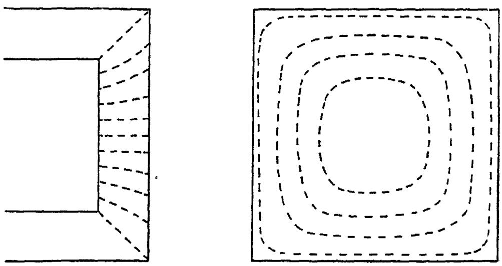
simplify the formulas, the following abbreviations for geometric ratios are introduced: $n = h/d_i$, where h is the inside length of the cylinder and d_i its inside diameter; and $j = b/a$, where a and b are two of the inside dimensions of the parallelepiped. The third dimension of the parallelepiped is the length equal to that of the cylinder ($h = n \cdot d_i$). As the ordinate in Figure 27, values are plotted for F', the ratio of area of the cylindrical furnace to area of the rectangular furnace.

Figure 28 compares the wall losses of cylindrical and rectangular furnaces of equal inside surface areas. Values for n are plotted on the abscissa and the ratio of the heat losses on the ordinate (R = losses of cylindrical furnaces/losses of rectangular furnaces). The various curves

are drawn for different values of the ratio $m = 2L/d_i$, where L is the thickness of insulation, equal for both the cylindrical and the rectangular furnace. It can be readily seen from Figures 27 and 28 that the cylindrical furnace gives smaller heat losses. It may be added that the closer a rectangular furnace approaches a cube or the cylindrical form of the condition $n = 1$ (equilateral cylinder), the smaller the wall losses become. A cylindrical furnace has smaller wall losses than a rectangular furnace of equal size because the heat insulation in the cylindrical furnace is utilized uniformly all over the circumference, while in the rectangular furnace the insulation in the corners is not fully effective.

Naturally, the selection of furnace shape does not depend on the question of heat losses alone. The heating time, spacing of the electrodes, etc. also play an important role. In this chapter however only the relationship of shape to wall losses is considered.

Selection of Insulation.—Equations (15) and (15a) make posssible the determination of temperature gradients throughout the insulation. The use of insulating material is limited by the maximum temperature to which the material may be exposed. This limitation is necessitated by reasons of mechanical strength. Obviously, the highest temperature



Cross Section Surface View

FIG. 29.—(At Left) Direction of heat flow in a furnace: in center of wall—perpendicular to surface; in the corners—diagonally; and between—the heat flow lines are as indicated. (At Right) Each of the dotted lines indicates an isotherm, the highest temperature occurring in the center, the lowest at the outside.

reached at any one point in a material is the decisive factor in determining the admissibility of this material. In a cylindrical furnace, the insulating material is uniformly heated over the circumference; surfaces parallel to the inside surface are isotherms. In rectangular furnaces, conditions are different: surfaces parallel to the inside surface are not isotherms; the insulating material in the center of each side of the furnace is heated to a higher temperature degree than that of the sides. Figure 29, showing a section of a furnace wall and a view of an outside sur-

face, illustrates this. The cross section contains the "heat-drop" lines and the view of the surface, the isotherms. It is obvious that the insulation of a rectangular furnace must be so designed that no point in the center is at a higher temperature than is permissible.

Figure 22 shows that, in general, materials for a higher operating temperature have higher thermal conductivities than materials used for lower temperatures. Consequently, they have less insulating value. Nevertheless, a layer of high-temperature lining must be applied all over the sides of the furnace in order to bring down the temperatures to a point permissible for a layer of high-grade low-temperature insulation on the outside. The necessary thickness of this inside protective layer is smaller in cylindrical furnaces than in rectangular ones. This fact is of very great importance. A thin inside layer means a small "inside area" for the following layer of high insulating value, and hence a larger thermal resistance. The conditions explained for single-layer insulation (comparison between rectangular and cylindrical furnaces) are still more accentuated for composite walls. It should therefore be accepted as a general rule that a cylindrical furnace shape always be considered as a possible substitute for a rectangular. (For the sake of completeness it can be stated that it would theoretically be still more economical to use spherical furnaces having uniform insulation so that the outside would also be spherical. However such shapes must be ruled out for practical reasons.)

Example.—Consider a rectangular furnace having inner dimensions of $8' \times 4' \times 4'$, an operating temperature of 2200 F and temperature of ambient, 70 F. The inside layer is 4.5-in JM 23 insulating firebrick; the outside layer is insulating block made as thick as a maximum permissible interface temperature of 1900 F will allow. Repeating the calculations given on page 39 results in the following: The maximum permissible thickness of the layer of insulating block is 5.6 in; choosing a round figure of 5.5 in, the heat loss is found to be 55000 Btu per hr; and 69.5 cu ft of insulating firebrick ($8'9'' \times 4'9'' \times 4'9''$—$8' \times 4' \times 4''$) and 113.5 cu ft of insulating block ($9'8'' \times 5'8'' \times 5'8''$—$8'9'' \times 4'9'' \times 4'9''$) are needed.

Compare this furnace with a cylindrical furnace of the same length (8 ft). The diameter of the furnace selected is larger than necessary (5 ft). The area of its base ($25\pi/4 = 19.6$ sq ft) is larger than the area of the base of the rectangular furnace (16 sq ft). Because of the steeper temperature gradient in a cylindrical shell, the thickness of the insulating fire brick on the inside need only be 4 in if the thickness of the layer of block of 5.5 in is to be maintained. With these values (4-in insulating fire brick, 5.5-in insulating block, temperature inside and outside the furnace the same as for the rectangular furnace) for the cylindrical furnace, the following results are obtained: The heat loss is 54000 Btu per hr; and 61.8 cu ft of insulating firebrick, $8'8''(5'8'')^2 \cdot \pi/4$—$8'(5')^2 \cdot \pi/4$, and 107.5 cu ft of insulating block $9'7''(6'7'')^2 \cdot \pi/4$—$8'8''(5'8'')^2 \cdot \pi/4$ are needed.

Thus the two furnace shapes will compare as in Table VI.

TABLE VI

EXAMPLE OF COMPARISON BETWEEN A RECTANGULAR AND CYLINDRICAL FURNACE

Dimensions and heat loss	Rectangular	Cylindrical
Inside volume, cu ft	128	156
Base area, sq ft	16	19.6
Thickness of firebrick lining, in	4.5	4
Thickness of insulation, in	5.5	5.5
Heat loss (steady-state), Btu/hr	55,000	54,000
Heat storage per unit volume, Btu/cu ft	428	345
Materials needed:		
firebrick, cu ft	69.5	61.8
insulating material, cu ft	113.5	107.5
firebrick, %	100	89
insulating material, %	100	95

Thermal Short Circuits.—Frequently it is necessary in furnace construction to have metal parts extending through the insulation. As explained previously (page 21), the effect of having two parallel paths for the heat flow—one through metal, the other through the insulation—is twofold: not only does metal cause a direct increase of heat flow because it has a much higher thermal conductivity (100 times or more than the insulation), but the path through metal also increases the heat flow through the neighboring insulation.

A systematic study of this problem has been published by Paschkis and Heisler.[21] The authors investigated the case in which strips of metal

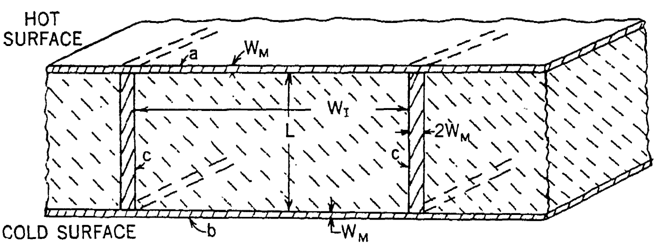


FIG. 30.—Arrangement of thermal short-circuits.

connect the inside hot surface with the outside cold surface, the latter being covered by metal. Only walls consisting of one material (single-layer walls) were considered. From the curves published, Figures 30, 31, 32, and 33 have been developed. Figure 30 shows the general arrangement under consideration. Figure 31 shows the heat loss for an

[21] V. Paschkis and M. P. Heisler, *Trans. Am. Soc. Mech. Engrs.*, 66, 653 (1944).

8-in thickness of insulation, a thickness of the metal, W_M, of $\frac{1}{16}$ in, a conductivity of the steel of 26 Btu per ft, hr, F, and a temperature difference of 500 F between the hot surface and the cold ambient. The

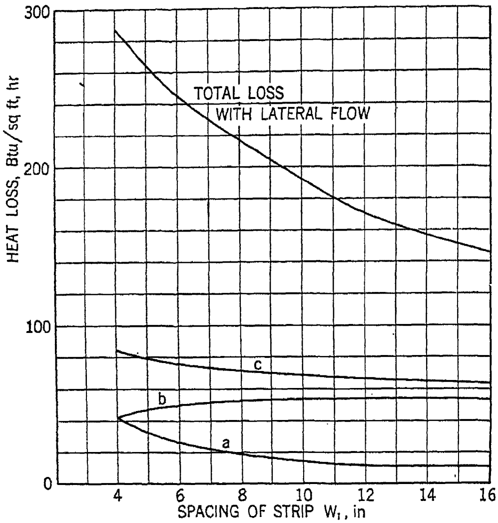


FIG. 31.—Increase in heat loss through thermal short-circuits. Curve *a*, rate of heat loss through insulation alone; curve *b*, rate of heat loss through steel strips alone; curve *c*, sum of curves *a* and *b*.

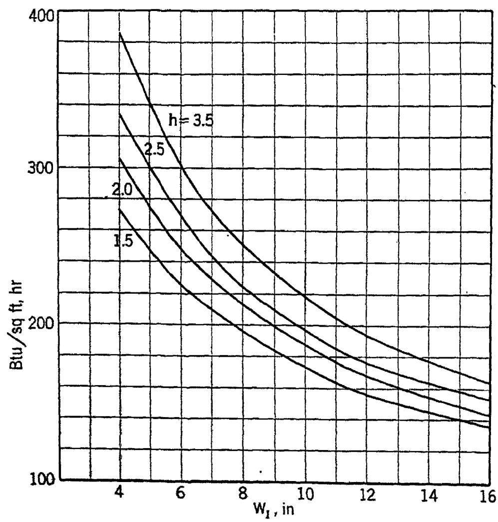


FIG. 32.—Increase in heat loss through thermal short-circuits. Influence of film conductance at various spacings of strips. Each curve is for a different value of film conductance, *h*.

film conductance on the cold side is 2 Btu per sq ft, hr, F. The spacings of the steel strips are plotted as abscissas, the rates of heat flow as ordinates. For the purpose of comparison, heat losses through the insulation alone (*a*) and those through the steel strip alone (*b*), as well as their sum (*c*), are plotted. By comparing these curves with *d* (which includes lateral heat flow) the influence of the lateral flow can be gaged.

Figure 32 shows the influence of the film conductance. The conditions are the same as those for Figure 31. The various curves show the rate of heat flow for different values of the film conductance.

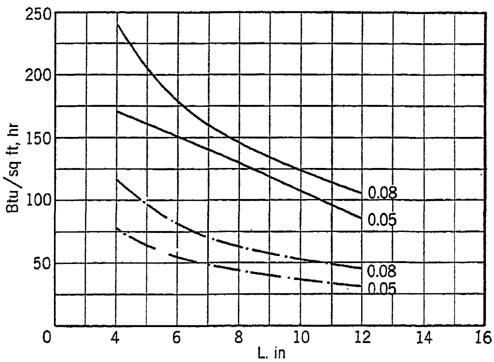


FIG. 33.—Increase in heat loss through thermal short-circuits. Influence of thickness of insulation (abscissa) and different thermal conductivities, *k*. Solid lines, heat loss with lateral flow; broken lines, heat loss without lateral flow.

Figure 33 serves as an example of the influence of thermal conductivity and thickness of insulation. The thickness of the insulation, *L*, is plotted along the abscissa, the rate of heat flow, *Q*, along the ordinate. Four curves are shown: two (broken curves) for the flow rate without lateral flow and two (solid lines) for the actual flow rate with lateral flow. Two curves are for a conductivity of $k_I = 0.05$, two for a conductivity of $k_I = 0.08$ Btu per ft, hr, F. With increase of thickness, the influence of lateral heat flow becomes smaller, particularly with higher conductivity (0.08) of the insulation.

A round steel rod of ¼-in diameter piercing an insulation of 9-in thickness and covered on both sides with steel sheets of 1/16-in thickness causes a higher heat loss than that which can be calculated from the straight flow through the rod. With a conductivity of the steel of 26 Btu per ft, hr, F and that of the insulating material of 0.1 Btu per ft, hr, F, a single rod causes a heat loss of 0.0175 Btu per hr, F. This figure is based on a film conductance on the cold surface of 2 Btu per sq ft, hr, F. If there were no lateral heat flow, the loss would be only 0.00018 Btu per hr, F. The lateral flow causes approximately a hundredfold increase

(0.0175/0.00018). If the temperature difference between the inside and outside of the furnace is 2000 F, the actual loss for each such rod is $2000 \times 0.0175 = 35$ Btu per hr.

(b) *Heat Losses in Intermittent Service* *

The heat losses in furnace walls discussed so far occur in the steady state, that is, after the wall has reached its final temperature at every point. This type of loss occurs almost exclusively in large furnaces operating 24 hours a day throughout the week, including weekends. However many furnaces, especially small ones, are heated up daily, operated for a certain length of time and then allowed to cool until they are again heated. The heat losses in such cyclic work were previously not calculable. The electric analogy method mentioned on page 31 made a rational analysis of the heat losses in intermittent service possible. A general solution of the problem is possible in the form of curves from which the heat loss for any condition can be read. For the purpose of a general solution, the heat loss, Q_i, occurring during a period or cycle, P, is compared with the steady-state heat loss, Q_{st}, that would occur if the furnace were operated over the entire period, P, at full temperature. The fractional heat loss, FHL, can thus be expressed by:

$$FHL = Q_i/Q_{st} \tag{24}$$

FHL is of course dimensionless, because Q_i as well as Q_{st} have the same dimension (Btu or kwhr). For the sake of general presentation it is further necessary to define the nature of the cycle by the intermittency, f, which is expressed as the ratio:

$$f = \frac{\text{time on}}{\text{time on} + \text{time off}} = \frac{\text{time on}}{P}$$

In this equation "time on" represents the total time during which the furnace is at full temperature and "time off," the total time between the end of the "on" period and the end of the cycle. The intermittency can have any value between zero and unity. If $f = 0$, then the furnace is never switched on and the heat loss Q_i is always zero. For $f = 1$, "time off" must be zero, the furnace is operated continuously, and the heat losses, Q_i, are then equal to the steady-state losses, Q_{st}, and $FHL = 1$.

* The material presented in this section is based on an exhaustive study made by the author for *Johns-Manville, Inc.*, on the heat loss of single-layer and double-layer walls in intermittent service. Permission to publish this part of the investigation is gratefully acknowledged. See also an abstract of this study by C. B. Bradley, C. E. Ernst, and V. Paschkis, "Economic Thickness of Insulation for Intermittent Operation," paper presented at the semiannual meeting of the ASME, Pittsburgh, Pa., June 19–22, 1944. See also *Trans. Am. Soc. Mech.*, 67 (1945).

Single-layer walls will first be considered. The length of the period and the design of the wall enter in the form of a third, dimensionless, characteristic figure, the dimensionless time P'. For the single-layer wall, that is, a wall constructed from one material only, P' is defined by:

$$P' = aP/L^2 \tag{25}$$

where a stands for the thermal diffusivity:

$$a = \frac{\text{thermal conductivity}}{\text{specific heat} \times \text{density}}$$

and L designates the thickness of the wall. If all items are introduced in consistent units, P' is dimensionless.

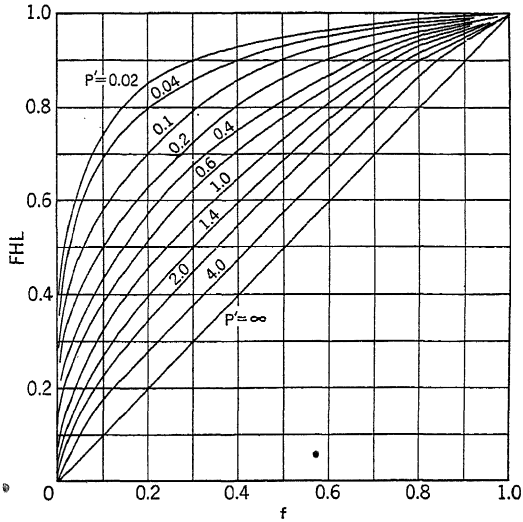


FIG. 34.—Fractional heat loss (FHL) as a function of intermittency (f); P', dimensionless time.

Figure 34 shows the values of FHL plotted against f for various values of P'. The chart is based on certain simplifying assumptions:

(1) Sudden temperature rise on the inside. At the moment the furnace is switched on, the inside surface reaches its final temperature as set on the temperature control.

(2) No film resistance on the inside or outside surface. The film resistance on the inside can usually be neglected; the influence of the outside film resistance should be allowed for by a correction factor, the size of which depends on the importance of the outside film resistance in relation to the thermal resistance of the wall.

(3) Empty furnace. There is no charge in the furnace during the heating-up period.

(4) Tight furnace doors. No heat escapes or is lost except from the outside surface of the furnace walls.

For $P' = \infty$, that is, for infinitely long periods, the influence of the heat stored at the beginning of the period is negligible compared with the heat loss during the "time on." Hence, for any value of f, $FHL = f$; the function of FHL against f is a straight line at a 45° angle. For $P' = 0$ (or more precisely, for P' approaching zero) the wall has no time to lose its heat content between two cycles. The heat losses are the same as in the steady state and $FHL = 1$ for all values of f.

(5) Infinitely large walls; no corner effects. In the case of small furnaces, it is necessary to make corrections for corner effects; the thermal resistance decreases and the thermal capacity of the wall increases.

Figure 35 shows a set of cross curves for Figure 34. P' is plotted as abscissa. Several curves for various values of f are shown.

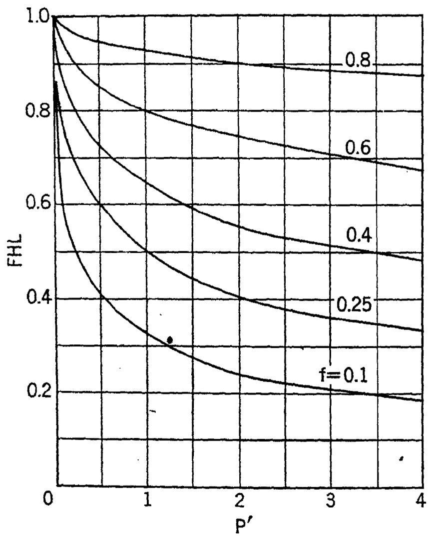


FIG. 35.—Fractional heat loss (FHL) as a function of dimensionless time (P'); f, intermittency.

For two-layer walls,[22] an approximate solution can be found by using an "equivalent single-layer wall" characterized by a fictitious thickness, L_r:

$$L_r = L_1 + L_2 \frac{k_1}{k_2} \tag{26}$$

[22] A method of reducing the complex double-layer wall to a single-layer wall was derived by Dr. M. Avrami, in the study made for *Johns-Manville, Inc.*

In Equation (26), L_r must be introduced together with the diffusivity, a, for the inside layer and with the actual length of the cycle, P'. L_1 and L_2 are the thicknesses of the inside and outside layers, respectively. The fractional heat loss found for this wall should then be corrected by a correction factor, which depends on the design of the wall as well as on the intermittency, f, and the film resistance on the outside surface.

Moreover, any two layer walls having equal values for the characteristic figures, x, τ, and g, behave in the same way. These figures are defined as follows:

$$x_1 = \frac{L_1/k_1}{L_1/k_1 + L_2/k_2}$$

$$\tau = \frac{k_1c_1\rho_1\left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right)^2}{P}$$

$$g = \frac{k_2c_2\rho_2}{k_1c_1\rho_1}$$

The following notations are used: L, thickness of layers; k, thermal conductivities; c, specific heat; ρ, density. Subscripts "1" refer to the inner layer and subscripts "2" to the outer layer.

(c) *Electrode Losses*

In the design of electrode as well as of resistor furnaces a problem of considerable importance arises in connection with the parts which serve to bring the energy into the furnace: electrodes in electrode furnaces and terminals of resistors in resistor furnaces. The problem is explained here for electrodes and applies equally to the terminals of resistors. It can be stated in general terms as follows: The electrode carries an electric current and, at the same time, a flow of heat not generated by the electric current flowing through the electrode but originating in the furnace. What, then, are the most desirable dimensions for the electrode? This question will be dealt with here in a rather general way because of the diversity of problems presented by the various types of furnace in which electrodes are used.

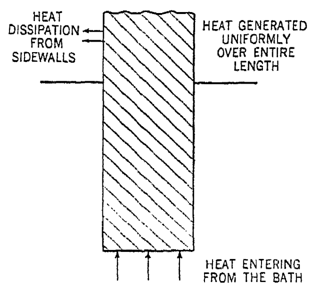


FIG. 36.—Schematic view of electrode.

Figure 36 illustrates schematically an electrode of an arc furnace. There are three different phases to be considered in the energy balance of such an electrode:

(*a*) Electricity causes ohmic losses (heat) at every part through which electric current flows. The generation of heat for a given current is smaller for larger diameters of the electrode. If the material has an

electric resistivity changing with temperature, the losses are not generated uniformly along the electrodes.

(*b*) Heat flows from the bath. This flow also represents a loss, because heat is withdrawn from the bath. The larger the diameter, the larger is the heat flow and the higher are the losses. If the thermal conductivity is not independent of temperature, the temperature gradient is not uniform over the length of the electrode.

(*c*) There is a radial heat flow. The electrode interchanges heat with the surroundings. Since, in almost every case, the temperature of the electrode is higher than that of the ambient, an additional heat loss occurs. The smaller the diameter of the electrode, the smaller is the loss. The radial heat losses are complicated by the fact that the various surfaces with which the electrode is in heat exchange are not equal in temperature and are, in addition, not of uniform material.

From the foregoing it appears there are two factors, one calling for a small electrode diameter and one calling for a large diameter. An optimum diameter giving the smallest over-all energy losses must therefore exist. The problem becomes more complicated by the fact that both the electric and the thermal conductivity change with temperature. The exact solution is not known. Hering[23] has derived the condition for smallest energy loss in the electrode under certain simplifying assumptions: neglecting the influence of radial heat flow, of the change of (electric and thermal) conductivities with temperature, and of the skin effect. While the energy losses are originated as heat loss (due to the temperature difference between arc and surrounding) and as electric losses (ohmic losses), they are carried away only as thermal losses: the ohmic losses generate heat, resulting in a temperature rise and consequently in heat losses. Hering shows that, for the end of the electrode next to the bath, three cases are possible:

(1) energy flowing from the electrode to the bath (besides, of course, the energy of the arc); this would involve a combined arc–resistance heating;

(2) no energy (besides that of the arc) flowing through that cross section; and

(3) energy flowing from the bath to the outside.

It can be shown by elimination that condition (2) yields the lowest energy losses. Neglecting the radial heat flow from the sides, let us consider Figure 37. If condition (1) were to prevail (energy flowing from the electrode to the bath), the temperature at some distance from the tip of the arc, say at sections $x \cdots x$, must be higher than at the tip of the arc. Then the heat flow to the cold end, which is, as stated above, the

[23] C. Hering, *Trans. Am. Electrochem. Soc.*, 16, 265 (1909).

only way through which the losses ultimately flow, is of course higher than in case (2): the length of the heat path from $x \cdots x$ to the cold surface is smaller than the length of the electrode and the temperature difference causing the flow to the cold end is higher than in case (2). In case (3), the energy which flows from the bath to the electrode must first flow through the electrode to the bath (arc). A higher energy would have to flow through the electrode than in case (2).

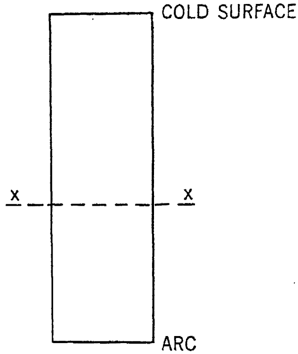


FIG. 37.—Electrode without lateral heat losses.

For the condition of case (2)—no energy flowing through the bottom tip of the electrode—Hering[23] has derived a formula showing the relationship between cross section, length, properties of the electrode, and bath temperature:

$$A = \frac{IL}{2}\sqrt{\frac{\rho}{k}\cdot\frac{1}{t_i - t_a}} \tag{27}$$

where A denotes the cross-sectional area of the electrode, I, the current carried by the electrode, ρ, the resistivity (electric), k, the thermal conductivity, $t_i - t_a$, the temperature difference between furnace inside and ambient, and L, the length of the electrode.

The above-mentioned assumption of no lateral heat flow is rather sweeping and greatly decreases the value of Hering's formula. Systematic investigations would be highly desirable.

B. Electrical

It is beyond the scope of this book to present complete instruction on the fundamentals of electrical engineering. Only a very brief review can be given, stressing such points as are of major importance in furnace work.

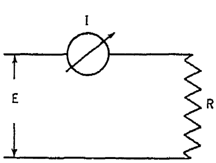


FIG. 38.—Single-resistor circuit.

1. Ohm's Law

If E denotes voltage (Fig. 38), I current, R resistance, and $1/R$ conductance, then in a circuit without inductance or capacitance:

$$E = IR \tag{28}$$

The resistance, R, of the resistor can be expressed by:

$$R = \rho \cdot L/A \tag{29}$$

where ρ denotes the resistivity, A the cross-sectional area, and L the length of the resistor. The over-all resistance of several resistors con-

nected in series (Fig. 39) equals the sum of the single resistances:

$$R = R_1 + R_2 + R_3 + \cdots \tag{30}$$

The over-all conductance of several resistors connected in parallel (Fig. 40) equals the sum of the single conductances:

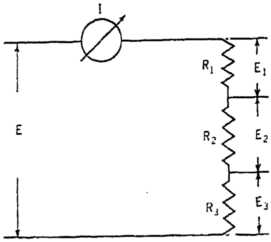


FIG. 39.—Several resistors in series.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \tag{31}$$

If E_1, E_2, E_3, etc. are the respective voltage drops of several resistors connected in series, then:

$$E_1 = IR_1; \quad E_2 = IR_2; \quad E_3 = IR_3$$

Consequently, the voltage drop in the individual resistors is proportional to their resistance:

$$\frac{E}{R} = \frac{E_1}{R_1} = \frac{E_2}{R_2} = \frac{E_3}{R_3} = I$$

If I_1, I_2, I_3, etc. are the respective currents passing through resistors R_1, R_2, R_3, etc. connected in parallel, then the total current, I, of the system is:

$$I = I_1 + I_2 + I_3 + \cdots$$

The amount of the currents is proportional to the reciprocal of the resistors through which the current flows (Fig. 40):

$$R_1I_1 = R_2I_2 = R_3I_3 = E$$

It should be noted that there is a perfect analogy between the equations for heat flow and those for the resistor circuits: compare Equations (1) to (3) on page 20 with Equations (28) to (31). Voltage is analogous to temperature difference; current is analogous to heat flow, and electric resistivity is analogous to the inverse of thermal conductivity.

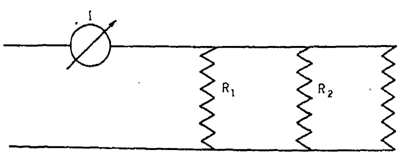


FIG. 40.—Several resistors in parallel.

This analogy goes even further. Electric capacity is analogous to thermal capacity (volumetric specific heat). This, together with the analogy for resistance, is the basis for the above-mentioned (page 31) Heat and Mass Flow Analyzer at Columbia University.

For alternating current, conditions are somewhat more complicated because, in most circuits, inductance and/or capacitance must be considered in addition to resistance.

2. Joule's Law

In a d-c or single-phase a-c circuit without inductance or capacitance, the power, W, is determined by:

$$W = EI \tag{32}$$

If, in three-phase systems, E_p is the voltage between one phase and the neutral, E is the voltage between two phases, and I is the current in one phase, then the power in one phase is given by Equation (32) using E_p for E. The power, W_3, in three phases together is

$$W_3 = 3\, E_p I \tag{33}$$

Also $E = \sqrt{3}\, E_p$ and therefore $W_3 = \sqrt{3}\, EI$. If Equations (28) and (32) are combined, W can be expressed in terms of I and R:

$$W = I^2 R \tag{32a}$$

Heat and electricity are both forms of energy. It is customary to carry out heat calculations in the English system, using the Btu as the unit of energy, and to carry out electric calculations in the electric system using the kilowatt-hour (kwhr) as the unit of energy (1 kwhr = 3413 Btu).

3. Power Factor

So far, only circuits having no inductance or capacitance have been mentioned. In furnace work, such circuits occur only in resistor furnaces; in this type of furnace, the inductance of the circuits can almost always be neglected. In direct-heat resistance furnaces, arc furnaces, and induction furnaces, inductance plays an important role. Inductance is of importance only in furnaces operated with alternating current. Alternating current and voltage oscillate between a maximum and minimum value with a frequency, f. (In most parts of the United States, $f = 60$; sometimes $f = 25$ is encountered. In Europe, $f = 50$ prevails with some use of $f = 42$.)

Inductance results in phase distortion. For a complete understanding of a phase distortion the reader must refer to textbooks in electrical engineering.[24] Here, only the elements which are of interest in furnace work will be briefly reviewed.

In Figure 41, voltage-time and current-time curves are plotted, times as abscissas and momentary values of voltage as ordinates. If this alternating voltage is applied to a resistor without inductance, a current curve, A, may result. The current is "in phase" with the voltage. If the current flows through an inductance having no resistance, the maxi-

[24] See, for example, F. W. Hehre and G. T. Harness, *Electrical Circuits and Machinery*. Vol. II, Wiley, New York, 1942.

mum of the current will occur later than that of the voltage (Curve *B*); this current is said to "lag in phase." If the current passes through a capacitance without a resistance, Curve *C* will result. The maximum of

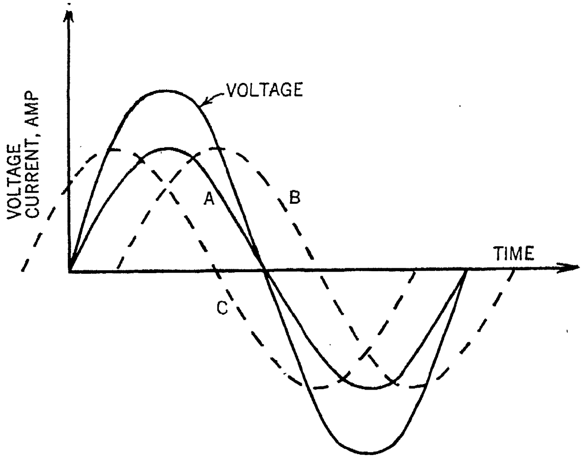


FIG. 41.—Current–time and voltage–time curves. Current *A* in phase; current *B* lagging in phase; current *C* leading in phase.

the current will occur earlier than that of the voltage. The current "leads in phase."

In order to determine how much the current and voltage are out of phase, the time of one whole period is divided in 360 parts. A purely inductive current has its maximum 90° earlier, a capacitive current 90° later than the maximum of the voltage. In these cases, the "phase angle" is 90°. If a current flows through a resistance *and* an inductance, a different phase angle will result. In this case, representation by vector can be used (Fig. 42). A line representing the voltage (to any appropriate scale) is drawn; the vector representing the current (to a different scale) is offset by the amount of the phase angle, φ. The total voltage drop for two elements in series, *e. g.*, resistance and inductance, can be divided into two parts, E_1 and E_2, which are offset 90°. E_1, representing the drop in the resistor, is in phase with the current. E_2, the drop in the inductance, lags behind the current by 90°. From Figure 42 it can be gathered that only a part, namely, $E \cos \varphi$, is in phase with the current. Only this part contributes

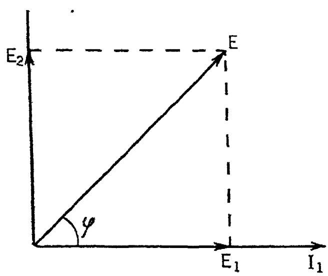


FIG. 42.—Vector diagram.

towards the power in the current. Therefore, in single-phase circuits other than pure resistance, the power is only

$$W = EI \cos \varphi \tag{34}$$

where $\cos \varphi$ represents the power factor.

In three-phase systems, the strict meaning of the power factor as explained above no longer holds. Only in a completely balanced three-phase system, in which the loads in all three phases are identical, is it possible to give one significant value to the power factor. Such a system is called "symmetrical." The power in asymmetrical three-phase systems is given by:

$$W_3 = \sqrt{3}\, EI \cos \varphi \tag{35}$$

In the case of unsymmetrical load, one can define an average power factor by:

$$\cos \varphi_{\text{av.}} = \frac{\text{sum of effective power in three phases}}{\text{sum of complex power in three phases}}$$

The sum of the effective power in the three phases can be determined by any of the usual measuring methods. The complex power (kvA) is the sum of the kvA input in the three phases, equal to the sum of the products of the amperage times the voltage to neutral (divided by 1000). The neutral point in unsymmetrical systems is off balance.

4. Skin Effect

For direct current, the resistance of a conductor is given by:

$$R = \rho \frac{L}{A}$$

where ρ is the resistivity of the resistor, A, the (cross-sectional) area of the resistor, and L, the length of the resistor. For direct current, the resistance of a body is inversely proportional to its area. This, however, is not true for alternating current. Alternating current causes around the axis of the conductor a magnetic field which in turn produces an electromotive force, which counteracts the original voltage and produces variations in current density. The current is forced towards the outside—the skin—of the resistor, leaving the core with low current density. This effect is termed the "skin effect."

For similar reasons neighboring conductors mutually influence their resistance. This mutual effect is called "proximity effect."

Skin effect and proximity effect increase with the current and with the frequency of the current. The calculation of skin effect is of special importance for arc furnaces, although it must also be considered in certain types of resistance furnaces. Methods of calculation and graphs will be found in the section on arc furnaces (page 143).

5. Induction

Alternating currents flowing in one conductor cause in neighboring conductors a voltage drop not in phase with the current, an inductive drop. It is possible to ascribe to a group of such conductors an "inductive resistance." Moreover, the current in each conductor can be considered as composed of a large number of individual current elements which act as though flowing in parallel conductors. Again an inductive resistance results. The inductive resistance of one conductor caused by a current in a neighboring conductor is:

$$r_i = 2\,\pi f M \tag{36}$$

where r_i is the inductive resistance, f, the frequency, and M, the coefficient of mutual induction. The inductive resistance of an inductor caused by its own current is:

$$r_i = 2\,\pi f L \tag{37}$$

where L is the coefficient of self-induction.

Calculation of the coefficients of self-induction and mutual induction of a circuit or a part of it is extremely difficult. Accurate calculations are possible only for geometrically simple arrangements.

An approximate knowledge of the inductive resistance is very important in all furnaces carrying heavy currents, especially in arc furnaces. Therefore a method of determining the inductive resistance of circuits will be discussed in the section on arc furnaces (page 149).

VII. FUNDAMENTALS OF FURNACE ECONOMY

A. Proportional and Nonproportional Losses

In almost all economic considerations, the different influence of proportional and fixed charges plays an important part. The difference between the two types has been repeatedly stressed. In the realm of heat (or fuel) economy a similar grouping can be observed. Here the difference between proportional and fixed charges are represented by the difference between proportional and nonproportional heat losses. Proportional losses are those which change with the load of the furnace; nonproportional losses are those which do not. The entire economic behavior of a furnace changes with the prevalence of one or the other type of loss. It is obvious, but can never be stressed enough, that, in a heat balance, a mere indication of losses without specifying what part is proportional is entirely unsatisfactory. Generally speaking, furnaces with predominantly proportional losses will be superior at low load, while furnaces with predominantly nonproportional losses will be superior at high loads.

B. Dependent and Independent Losses

There are a number of furnace types in which an increase in one type of loss automatically brings forth an increase in another type. Losses are called "dependent" if their amount depends on that of other losses and "independent" if their amount does not depend on that of other losses.

In the case of dependent losses, it is obviously useless to write a heat balance in the form of a mere addition. A change in one item (*e. g.*, heat losses in the arc furnace) would not change the useful heat by the same amount; other losses would also change. This fact is very frequently overlooked, and makes many published heat balances useless.

A written heat balance involving dependent and independent losses might appear as follows:

Useful heat (at full load; specify temperatures)	65%
Proportional losses in the connections as 10% of the energy involved	6.5%
Wall losses (independent)	23
Corresponding dependent losses as 10%	2.3%
Core losses in transformer (independent). These are influenced, not by the current drawn from the transformer, but only by the voltage, which does not change with load	3.2%
Total	100%

A graphic presentation of a heat balance is shown in Figure 43. The conventional method is shown in Figure 44.

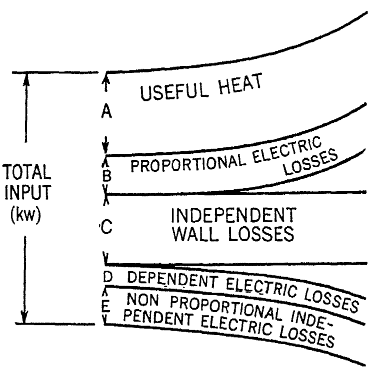


Fig. 43.—Energy balance showing proportional and dependent losses.

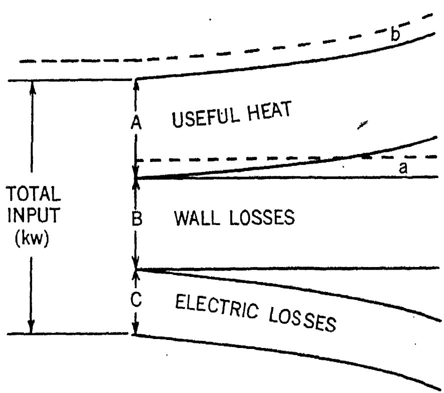


Fig. 44.—Energy balance, conventional type.

One very striking example may be found in arc furnaces. The current flowing in the transformer, busses, and electrodes causes energy losses. The current must be large enough to cover (at the given voltage) the heat demand of the furnace. Now if this heat demand is increased

by an increase of heat losses (*e. g.*, through change in lining, from keeping the doors open for a longer time, etc.), the total increase in energy will be larger than that indicated by the increased heat demand alone. The greater heat demand calls for larger current, which in turn causes higher electric losses. The losses in the lining in this example are independent losses, while the electric losses are dependent.

Similar combinations occur in other types of furnaces. In electric furnaces the dependent loss usually is an electric loss. An example in which dependent and independent losses are heat losses is given in Figure 45, in which a bar is shown extending out of a heat-treating furnace chamber, *B*. The end in the furnace is at an elevated temperature. The extending end of the bar (*A*) causes (independent) heat losses because it is cooled outside the furnace. The temperature in the furnace, and consequently the (dependent) wall (*C*) losses, must be higher because the protruding bar end causes heat losses. An increase of loss from *A* (as through an air-stream cooling, or through greater length or cross section of *A*, etc.) calls for a higher temperature in *B*, which in turn results in higher wall losses through *C*.

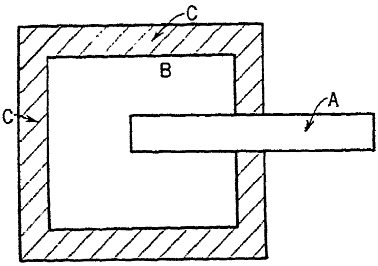


FIG. 45.—Heat loss as dependent losses.

TABLE VII
EXAMPLE OF ENERGY BALANCE

Factor	Per cent	Case *A*, kw	Change	Case *B*, kw	Per cent
Total input		3000	—	3000	
Useful heat	60	1800	−220	1580	52.67
Proportional electric losses	6	180	− 22[a]	158	5.27
Independent wall losses	29.34	880	+220	1100	36.67
Dependent electric losses	2.93	88	+ 22[a]	110	3.67
Nonproportional independent electric losses	1.73	52	—	52	1.73
TOTAL	100.00	3000		3000	100.00

[a] These two items cancel, since the drop in useful heat is just as large as the increase in wall losses.

Incidentally, in all fuel-fired furnaces, a combination of dependent and independent heat losses is regularly encountered. The losses due to incomplete combustion and the stack losses are both in part proportional losses (proportional to the useful heat) and in part dependent losses (dependent on the wall losses). If the wall losses change (different insulation, etc.), stack losses and losses through incomplete combustion change also.

The diagram in Figure 44 conveys the idea that an increase, for example, in wall losses, as shown by dotted line *a*, should increase the total heat only to an extent indicated by dotted line *b*. From the above discussion it is obvious that the increase will be higher because the electric (dependent) losses increase. In this case, the input was not kept constant. If the total input is kept constant and the (independent) wall losses change, then the useful heat changes only by the same amount. The electric dependent losses change too, but the proportional losses change in the same amount. The percentage of course will change. An example is given in Table VII. If the useful heat and not the total input had been kept constant, the increase in wall losses (220 kw) would have caused a greater increase (220 + 22 kw) in the total losses and therefore in the input, and 3242 kw (3000 + 220 + 22), not 3220 kw (3000 + 220), would have been necessary.

C. Cost Comparison between Electric and Fuel-Fired Furnaces

Discussions about the application of electric furnaces usually begin with a comparison of operating costs in electric furnaces *vs.* those in fuel-fired furnaces. Frequently, though unjustifiably, a comparison is made only between the heating costs, all other costs being neglected.

It is impossible to give any fixed ratio of fuel cost between an electric furnace and a fuel-fired furnace. Not even a general ratio of heat consumption can be given. Generally speaking, however, it can be said that an electric furnace has a smaller amount of proportional losses than a fuel-fired furnace used for the same purpose. Hence, from the viewpoint of heat consumption, the electric furnace becomes more advantageous with decreasing furnace output. This is due to the different importance of proportional and unproportional losses in electric and fuel-fired furnaces. Examples of proportional losses in fuel-fired furnaces are part of the stack losses. (The rest of the stack losses are dependent losses; see above.) Examples in electric furnaces are the losses through electrodes. Nonproportional independent losses in both types of furnace are the losses from the furnace walls.

An empty furnace, held at constant temperature, causes "idling" losses (see page 3). These losses are higher in fuel-fired furnaces than in electric furnaces because of that part of the heat in the waste gases (sensible heat and heat of incomplete combustion) due to the fuel, which is used to cover the wall losses.

Proportional losses play a much more important role in fuel-fired furnaces than in electric furnaces. In Figure 46, the furnace output (in any unit: weight, number of pieces, or volume per hour) is plotted as the abscissa; on the ordinate is shown the heat consumption in Btu per lb.

One curve is for electric furnaces, the other for fuel-fired furnaces. As the output approaches infinity, the heat consumption in Btu per lb will approach the theoretical value (*i. e.*, the value with no heat losses) in an electric furnace without proportional losses; in a fuel-fired furnace, the heat consumption will equal the theoretical value divided by the factor of proportional losses (*e. g.*, the efficiency of combustion). For very small output, the higher idling losses of a fuel-fired furnace make themselves felt. Consequently, the two curves of heat consumption per unit are closer for high output than for low output values.

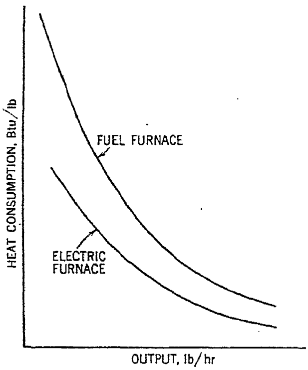


FIG. 46.—Specific energy consumption *vs.* output.

The superiority of the electric furnace as far as heat consumption is concerned is more marked for low output, and consequently electric furnaces have a better chance of competing with such fuel-fired units on a heat-cost basis. Suppose, for example, that a furnace, after the load is brought up to temperature, must be held for a long time at temperature (*e. g.*, case-hardening furnaces; annealing of malleable iron in batch-type furnaces). Then the lower idling losses of the electric furnace during the long holding time may be of decisive advantage.

For the other cost items, reference is largely made to the comparison of the advantages and disadvantages of both systems (electric and fuel heated, page 13).

If the number of rejections is decreased by the application of electric heat, the actual savings are obviously larger than those expressed by the elimination of repeated heating. All cost items entailed in the manufacture of the piece prior to the heating process are also saved and would, in a cost balance, have to be credited to electric heat. Suppose that a piece must be rejected because of heat treating, and that before heat treating it underwent operations totalling \$A. If through some new process n fewer such pieces are rejected, then the saving of $(n \times A)$ dollars should properly be credited to the equipment making this saving possible.

D. Comparison of Various Types of Electric Furnaces

It is common practice to talk of the "efficiency" of a furnace as being a certain percentage. This usage is very misleading and should be abandoned. The efficiency of a furnace will be entirely different for different processes, even if all are carried out at the same temperature.

The "efficiency" will also be different if the furnace is used for the same operation, but once in continuous service and once in intermittent service. The efficiency will change with different ways of loading, with different duration of open-door periods (depending perhaps on the skill of the operators), and on many other items. The same criticism naturally holds for other derived parameters, such as "specific power consumption" and similar well- or poorly-defined terms.

An "efficiency" figure, in order to be definitely defined, would have to be vested with so many restrictions and conditions that it would become practically inapplicable. Wherever possible, "characteristic figures" are preferable. They should include all figures which are necessary to calculate the energy consumption and the behavior of the furnace under various conditions. The nature of these characteristic figures varies for the different types of furnaces and will be discussed in the respective chapters. Among the characteristic numbers, energy losses, broken down into proportional and nonproportional, dependent and independent, and connected load are of paramount importance.

E. Economics of Furnace Operation

A furnace producing a product of a desired quality and quantity can show entirely different economy in different plants or in the same plant on different days. In other words, the same furnace can be operated with more or less expense, always yielding the same result in quality as well as in quantity. Some of the common rules resulting in low cost are discussed below. They can be classified into two groups: means of decreasing the power consumption and means of decreasing the power cost.

In order to decrease the *power consumption* for a given production, the following steps should be taken:

(*a*) Avoid additional heat losses, such as may result from keeping doors or covers open longer than absolutely necessary.

(*b*) As far as possible avoid temperature changes in the furnace. If, in a furnace, different operations calling for different temperatures must be carried out, two ways of operation are possible: The loads with different temperatures are put through the furnace either promiscuously or in an orderly manner. Putting them through in an orderly manner, so that as many operations at the same temperature as possible follow each other, often results in remarkable savings. Heat storage in the walls and heat losses in waiting for cooling are saved. It is desirable to run first as many loads at high temperature as possible. The heat storage in the wall may often be partially recovered during the first of the loads at lower temperature. If the loads at low temperature occur before a shut-down (week-end), additional savings will be made. The idling losses during the shut-down take place at lower temperatures.

(*c*) Utilize the furnace in time and volume as far as possible. With small loads, the specific power consumption increases rapidly because the majority, if not all, of the losses are independent and not proportional.

Decreasing *power cost* is possible only with certain types of power rates, which depend partly on consumption and partly on demand. Simple power factor clauses should also be mentioned here. In certain instances, operating to give minimum power cost will result in higher power consumption. A factory working during one or two shifts pays the demand charge for the highest demand during these hours. During the third (or second and third) shift, the demand is very much lower. Electric furnaces can be conveniently operated so as to have their peak load during the night hours.

For instance, for a plant having a maximum demand of 2000 kw, and for which during the night shift the demand is only 1600 kw, erection of an electric furnace is contemplated having a connected load of 1000 kw. The peak load would be operated only for four hours; then the load would drop off and remain low, perhaps at only 200 kw. The low load would be maintained for 15 or 16 hours. In such a case, it would be desirable to decrease the connected load to 600 kw. The total load of the factory would then increase from 1600 kw to $1600 + 600 = 2200$ kw. The day load would increase from 2000 kw to $2000 + 200 = 2200$ kw. Because of the smaller connected load the heating would take seven to eight hours instead of four hours. The power consumption would be higher; but the demand charge would increase only for the 200 kw instead of for the 600 kw which would result from a furnace input of 1000 kw superimposed on the 1600-kw night demand.

In almost every case, a decrease in the connected load will result in an increase of power consumption. The gain and loss must be weighed against each other and the outcome will depend on the rate scale.

The notations are as follows:

Q_i = rate of axial heat flow to the element from the preceding element; Q_g = rate of heat generation in the element; Q_o = rate of axial heat flow from the element to the following element; and Q_L = rate of radial heat flow from the element to the surrounding charge. Q_i, Q_g, and

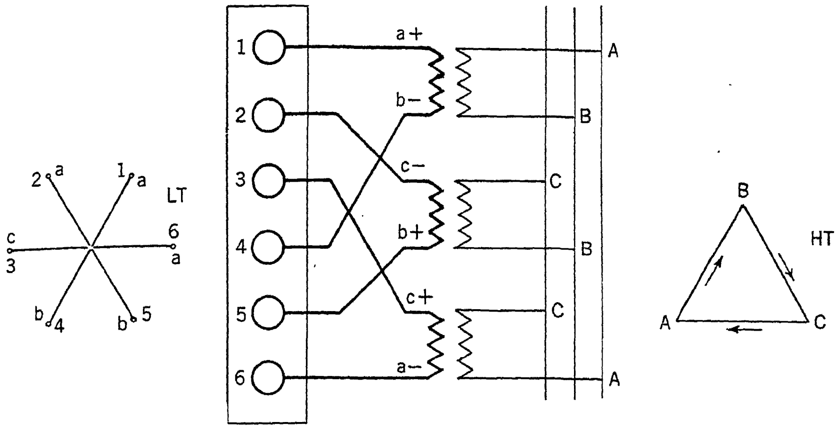


FIG. 154.—Connections for six-electrode furnaces (*HT*, high tension side; *LT*, low-tension side).

Q_o could be calculated as functions of temperature if the thermal and electric conductivities of the electrode were known. Q_L depends on the temperature of the charge surrounding the electrode, on the thermal conductivity of the charge, and on the boundary resistance between electrode and charge. Matters are complicated by the fact that the temperatures across the electrode are not uniform.

Calculations are almost hopeless. One possible way of approach would be the electric analogy method (page 31).

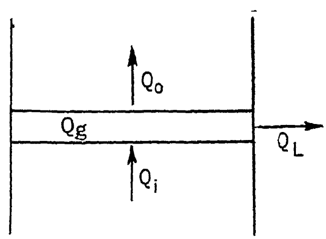


FIG. 155.—Energy flow in an electrode element.

By making a number of assumptions, a calculation was attempted for a carbon electrode of 30-in diameter, carrying 30,000 amp. The calculation was made in order to determine the temperature distribution over the length of the electrode and the amount of axial heat flow. The result is shown in Figure 156. These curves are based on the following assumptions:

(1) A uniform temperature exists over the cross section of the electrode at any level.

(2) The thermal and electric conductivities of the electrode are independent of temperatures; electric resistivity = 21.6×10^{-3} ohm × sq in per ft, and thermal conductivity = 3.87 Btu per ft, hr, F.

(3) The temperature of the charge facing the electrode tip is 550 C.

(4) The temperature of the charge at charge level is 52 C.

(5) There is a linear temperature drop in charge between the two levels mentioned in (4) and (5).

(6) The electrode is immersed 2.5 ft.

(7) The length of the electrode from above the charge to the lower end of the holder is 3 ft.

(8) The electrode holder is 1 ft long.

(9) There is no temperature drop in the charge in a radial direction.

(10) The boundary resistance between electrode and charge is 11 Btu per sq ft, hr, F.

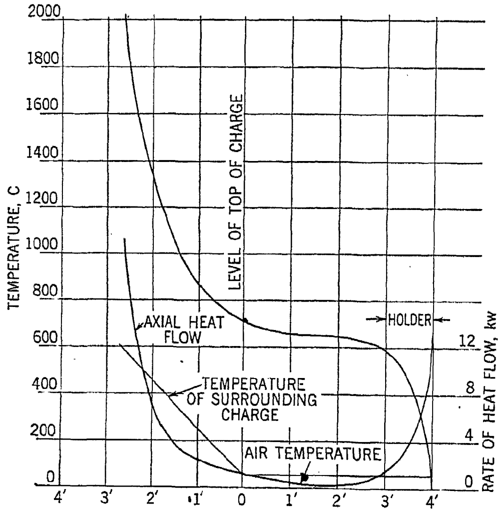


FIG. 156.—Typical temperature distribution and heat flow in an electrode.

Obviously some of these assumptions are quite sweeping, and thus the value of the curves is more of an indicative nature.

From the calculations, it becomes evident that the value of the temperature facing the tip of the electrode assumed under item (3) is of paramount importance. For instance, an assumption of 805 C instead of 550 C leads to an electrode temperature at the tip of 3225 C instead of 1775 C.

These approximate calculations seem to indicate that only very little heat flows axially through the electrode at the top level of the charge. Almost all heat generated below the charge level flows through the perimeter of the electrode back to the charge and is thus used for preheating. The temperature drop in the electrode is very steep. If the end of the electrode extending into the charge becomes too big, the temperature of the electrode tip exceeds safe limits, and burning of the electrode results.

The greater the depth of immersion of the electrode the smaller the permissible electric load of the electrode becomes. In a specific case, it was found that the current for a 30-in carbon electrode had to be reduced to 19,000 amp because the depth of immersion was 10 ft.[7] In shallow furnaces, the safe current-carrying capacity is 30,000 to 32,000 amp (30-in electrode).

From similar considerations and the same assumptions as those used for Figure 156, Figure 157 was drawn. It shows the decrease of per-

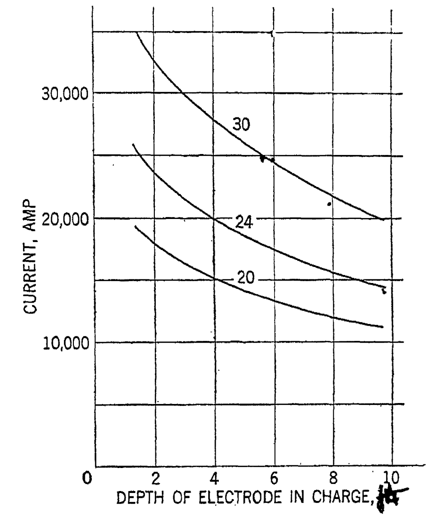


FIG. 157.—Permissible electrode current. The numbers on the three curves indicate the diameter of the electrodes in inches.

missible current (ordinate) with increasing depth of immersion (abscissa) for various electrode diameters. This graph is again to be regarded as indicative rather than as giving values which can be used directly.

E. BUSSES

The heavy current necessary in ferro-alloy furnaces makes the design of the bus system particularly important. In increasing the cross section of the copper in order to lower the ohmic losses, a point may be reached at which the savings in ohmic losses would be outweighed by the expense of increased weight of copper, insulation, etc. A curve might be drawn showing the annual cost of the ohmic losses and the carrying charges plus depreciation cost, both plotted *vs.* current density. An optimum density might thus be determined. This process would have to be repeated for various designs, permitting different current densities, in order to find

the most economical solution. It should, however, be kept in mind that the losses in the busses and the voltage drop determine the possible output. In most instances, the ohmic loss is so small compared with the inductive loss as to be negligible.

F. Transformer

Transformers for ferro-alloy furnaces are generally custom built. They are characterized by their heavy low-tension coils and leads, which must be very firmly braced. If only one product is manufactured in a furnace and the voltage requirements are well known, a range of six taps, two below and four above normal voltage, about 5% apart, will be sufficient. If the product is new or if several products are manufactured, the range of voltages must be large. The lowest taps can be specified for reduced capacity.

The changing of taps is generally carried out with the power off. However, for certain products, for example, silicon carbide and carbon reduction of magnesium, it is necessary to be able to change the taps under load.

Since the reactance of the furnace leads is relatively high, the reactance of the transformer should be as low as design permits. The reactance is generally given in relation to the highest tap. In ohmic value, it changes very little for the different taps. In a 6000-kva transformer with a high tap of 130 v, a 3.5% reactance will result in an inductance of 0.000095 ohms, which will be about 10% of the total furnace reactance.

The number of low-tension coils will depend upon the voltage and the capacity of the transformer.

For reasons of flexibility it is advisable, in a three-phase transformer, to provide the high-tension side with a terminal board with delta and star connections.

A three-phase furnace can be operated either from a three-phase transformer or from three single-phase transformers. Because of the spare transformer, necessary to safeguard production, the number of furnaces operating in a plant will decide which way of feeding (single-phase or three-phase transformers) is more economical. The price of three single-phase transformers is approximately 1.25 that of one three-phase unit of the same total output. Considering the presence of one transformer (single-phase or three-phase) as spare as sufficient, single-phase and three-phase transformers will have equal first cost, if 2.33 furnaces are in operation. In other words, with two furnaces, the single-phase transformer is less expensive; while for three or more, the three-phase transformer is less expensive. (This is only an approximation because of the possible lower price for many single-phase transformers.)

Chapter Two

Electrode Melting Furnaces

(*Arc Type and Arc Resistance Type*)

From the electrical viewpoint, arc resistance furnaces, arc furnaces, and direct-heat resistance melting furnaces are so closely related that they can be treated as one type. In respect to design, however, there are large differences.

Compared with the other types, indirect-arc furnaces (Fig. 3, A_2, page 5) are of relatively little industrial importance, their chief application being in the melting of small batches of iron, steel, and copper and other nonferrous metals.

Direct-arc furnaces—without any resistance component—are used only in chemical processes for the heating of gases to high temperatures. As a typical example, the Haber process for the fixation of nitrogen may be cited. The design of direct-arc furnaces is entirely different from that of other arc furnaces and will not be discussed in this book. Relatively high voltages (some thousand volts) are used. There are several special articles available on the subject.[25]

The largest group of melting furnaces, which are of combined type, may be called arc resistance furnaces (Fig. 3, A_2–$B_{1b\beta}$). In this group may be found all variations, from the Heroult type, used for melting iron and steel, in which practically all of the energy is produced in the arc and almost none in the resistance of the bath, to ferro-alloy melting furnaces, in which the arc component is almost negligible, the greatest part of the energy being transformed into heat in the resistance of the melt.

From the electrical viewpoint, these various types are identical in principle, differing at most only in some minor degree, *e. g.*, in the design of the busses. The mechanical design is, however, quite different. Some furnaces are "open," that is, they operate without cover and a cold charge surrounding the electrodes and covering the melt guards against excessive radiation. Other furnaces have covers. Some furnaces are tiltable; others are far too large to be moved. Some furnaces are arranged for continuous operation; others are emptied after melting each

[25] H. Pauling, *Elektrische Luftverbrennung.* Knapp, Halle, 1929.

charge. There are many other mechanical considerations which influence the design of furnaces, and all will be considered in detail in the section covering furnace design (pages 78–164). Table VIII reviews briefly some of the more important industrial applications of electrode melting furnaces. Electrochemical processes are included for the sake of completeness, but it should be noted here that such processes use the electric energy for chemical reactions in addition to the generation of heat, and that their design is therefore subject to different rules.

TABLE VIII

INDUSTRIAL USES OF ELECTRODE MELTING FURNACES

Type of furnace	Process and product
Direct-arc	Production of nitrogen Production of cyanic gases
Indirect-arc, direct-resistance	Nonmetallic materials: Corundum Molten concrete Calcium carbide Phosphate Slag (for production of slag wool) Smelting: Iron ore Copper ore Ferro-alloys: chromium, manganese, molybdenum, silicon, tungsten, vanadium, boron Melting: Iron and steel, nickel
Indirect-arc	Melting: High-alloy steel Iron, especially in foundries Brass Bronze
Electrolytic cell	Production by electrolysis of: Aluminum Magnesium Calcium Sodium Lithium

Pages 78–207 of this chapter will deal mainly with the covered type of arc resistance furnaces and (briefly) with indirect-arc furnaces, both used for the melting of iron and steel.[26] On pages 165–207, the operating and efficiency diagrams will be discussed. Pages 207–225 will cover ferro-alloy furnaces.

I. INTRODUCTION

A. SUMMARY

Heat is generated in an arc furnace by the passage of an electric arc either between two electrodes or between one or more electrodes and the charge. Hence, generally speaking, the arc furnace consists (Fig. 47) of a furnace chamber and two or more electrodes. The furnace chamber

[26] An extensive review of present-day practice of arc furnace design and operation, including metallurgical aspects, is contained in an article by T. J. Ess, "The Modern Arc Furnace," *Iron Steel Engr.*, 21, AF7 (1944).

has a lining suitable for the material to be heated and for the operating temperatures. The lining is contained within a steel shell which, in many cases, can be tilted or moved. Generally the chamber is built of two parts: for arc resistance furnaces, the roof is detachable; only in the case of the rocking indirect-heat arc furnace is no part of the chamber normally detachable.

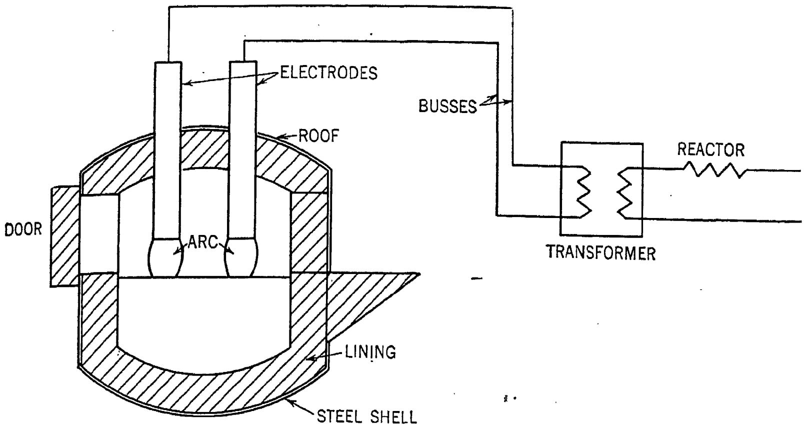


FIG. 47.—Arc furnace.

The electrodes are usually not connected to the line voltage directly, but through a transformer, partly because the furnace design calls for unusual voltages, partly because the voltage must be changed during operation, and finally because the transformer tends to stabilize the arc. Special connectors are required between the electrodes and the transformer because the furnaces operate on relatively low voltages and therefore use extremely large currents. The position of the electrodes determines the length of the arc, which in turn influences the current and power. The position of the electrodes is controlled by automatic devices, which maintain either the current or the power at a set value.

Much of the design of arc furnaces is empirical; at least no systematic investigations such as have been made on steam-generating units have been published to justify the empirical selection of dimensions. For each major part of the furnace design, the problems involved will be reviewed, analytical answers will be given as far as possible, and reference will be made to practice for the rest.

B. BATH VOLUME, HEARTH VOLUME, OUTPUT, AND CONNECTED LOAD

In general, furnaces should be designed for a given output. The output is a function of the size of the furnace chamber and the melting

time. The "bath capacity," expressed in tons, can be found if the desired furnace output (weight per unit of time—ton per hour [27]) and the melting time are known:

$$\text{bath capacity} = \text{output} \times \text{melting time}$$

The bath volume follows from the bath capacity. If the charge consists of low-density scrap, the hearth volume must be made larger than the bath volume. If metallurgical reasons call for a refining period after the completion of the melting, the total heating time (melting and refining) should be introduced rather than the melting time alone. If the heating time is known, the furnace (holding) capacity results from the desired output. In times of pressing production needs, the output is frequently increased beyond the rated value, at the expense of efficiency and sometimes even of quality. This will be discussed later.

Shortening the heating time results in lower cost: first cost of the furnace, depreciation, labor, and cost of energy per ton decrease with decrease of heating time. It is thus important to find a design and rating giving the shortest possible heating time, since the *refining* time usually cannot be changed. (Refining in an acid furnace requires 0.5–1 hour, in the basic furnace approximately 1.5–3 hours. For an explanation of "acid" and "basic" furnaces, see page 89.) Shortening of the *heating* time can be brought about only by a decrease of the melting time. To melt a given amount of material, a fixed amount of heat—expressed as Btu or kwhr—is required. A shorter heating time therefore means a greater flow of heat, expressed as Btu per hr or kw. Such an increased flow can be achieved by one of two means: either by decreasing the losses per unit of time, or by increasing the connected load (the latter being the rate of energy input at the rated voltage). With constant connected load, a decrease of losses per unit of time leaves more energy free for useful heat. An increase in connected load, if not accompanied by an increase in losses, yields more useful heat.

The possibility of decreasing the losses per unit of time is limited by difficulties of design and material and will be discussed later. The increase of connected load deserves careful attention. In fact, in the 1920's, there was a time when arc furnaces were built with increasingly higher connected loads. The very great increase in connected load in recent years has brought to the fore the danger of overheating the material under the electrodes. The problem of transporting the heat away from the zone of generation arises. Little is known of the apparent

[27] Until recently, ingot mills generally expressed their output in "long tons," while foundries expressed their output in "short tons." One long ton equals 2240 pounds, or almost exactly one metric ton. One short ton equals 2000 pounds. In earlier publications, these different usages may be confusing.

thermal conductivity of the charge or of the mechanism of heat transfer in the bed of broken solids (charge) interspersed with liquid steel.

Means have been sought to counteract this danger (pages 77–78). In medium-size and small furnaces this limitation does not occur, although other questions do arise: Where is the limit, if overheating of the charge is not to be considered? How far can the connected load be increased? What are the limitations to such an increase? In the indirect-arc furnace (Fig. 3, A_2, page 5), as well as in the more frequently applied combination of indirect-arc, direct-resistance furnace (Fig. 3, A_2–$B_{1b\beta}$), heat is generated locally in the arc. This is obvious in the case of the indirect-arc furnace; but for the combined arc–resistance furnace, the main transformation of energy takes place in the arc and only a small portion in the bath. In electric arcs as used in arc furnaces, a large amount of energy is produced in the arc proper, while in the two ends only 10–15% of the energy is transformed. Changing the melting time therefore means changing the power in the arc. This can be brought about by changing the voltage or by changing the current. Increase in voltage with current unchanged calls for increasing the length of the arc. Increase in current with voltage unchanged is obtained by shortening the arc. However, there are limits to increasing the voltage and current of the arc: high voltage with the resulting long arcs cause rapid wear of the roof of the furnace. Furthermore, potentials of more than 325 v in the furnace are considered undesirable because of possible danger to the operator.

High current makes special precautions against losses in the busses necessary, can cause difficulties in the (electrical) operation, and causes higher electric losses in the busses. Also, the necessary thickness of the electrodes increases with the current. Thick electrodes cause higher heat losses than thin electrodes, and are more expensive to replace. Since, in general, these considerations are not reducible to mathematical formulas, Figure 48 is reproduced, based on empirical data showing the connected load per ton capacity (kva per ton) plotted *vs.* furnace capacity (tons of steel). The curve is of an indicative nature only and allows for considerable margin. Connected loads as high as 600 kva per ton are found in small furnaces, and loads as high as 240 kva per ton are not uncommon in large units. The limit to the power which can be carried safely by one group of three electrodes is generally considered to be 15,000–18,000 kva.

In small furnaces, the melting time amounts to approximately 1.6–2 hours, although melting times as low as one hour have been reported. This does not take into consideration time for refining, which depends on metallurgical factors entirely independent of the furnace size. In large furnaces, the melting time is higher and reaches approximately 3–5 hours

for the largest size of furnace. If the heat and electric losses for a given furnace size can be estimated, the influence of the connected load on the melting time can be determined as follows: by subtracting the losses in kw from the connected load, and dividing the theoretical power consumption for the total furnace capacity (kwhr) by the remaining load (connected load minus losses). An exact calculation of the theoretical power consumption/useful heat would have to be based on the weight of the charge, the specific heat, heat of fusion, and desired temperature. Inasmuch as the specific heat of various steels, particularly also in the liquid state, are not known accurately, it is customary to take an over-all figure of 0.154 kwhr per lb for steel.

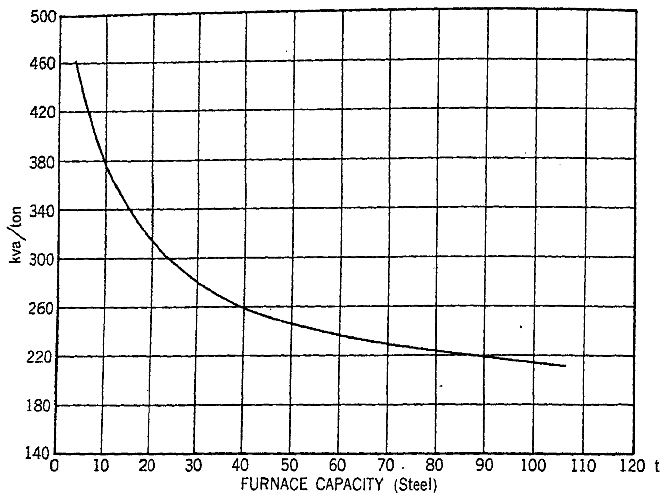


FIG. 48.—Connected load *vs.* bath capacity.

Some approximate figures for these constants are shown in Appendix C. The method is obviously only an approximation because part of the losses are proportional (page 64).

Decreasing the heating time by using higher connected loads results in higher first costs and a higher demand charge in the power rate. Obviously such higher costs can be appealing only if the output is increased substantially. Furnaces with long refining periods will have connected loads that are small in relation to their holding capacity. Here, an increase in connected load would not pay because the total output would not decrease proportionally to the melting time; in other words, for a certain connected load the bath volume will be larger for furnaces with long refining periods. Under emergency conditions an increased output is obtained by increasing the bath volume at the ex-

pense of thermal efficiency. The larger volume of steel must still be held for the same refining time.

The nature of the charge also influences the optimum value of connected load. Bulky pieces in the charge justify a higher connected load than light turnings.

An increase of connected load without further precautions causes a shorter melting time, but at the same time results in greater temperature differences in the bath. In order to avoid such differences and to secure more uniform temperature in the charge, the following means have been recommended:

(*a*) Generate part of the heat in the charge. This calls for bottom electrodes, which have been tried and proved to be not very successful. They cause considerable complication in design; and their results as far as temperature uniformity is concerned are limited because, if the bath resistance is in series with the arc, the major part of the energy is transformed in the arc. Figure 49 is a diagrammatic sketch of such a furnace.

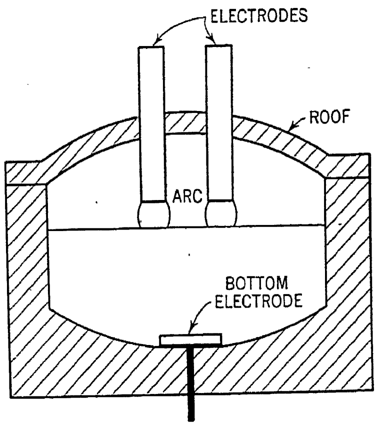


FIG. 49.—Furnace with bottom electrodes.

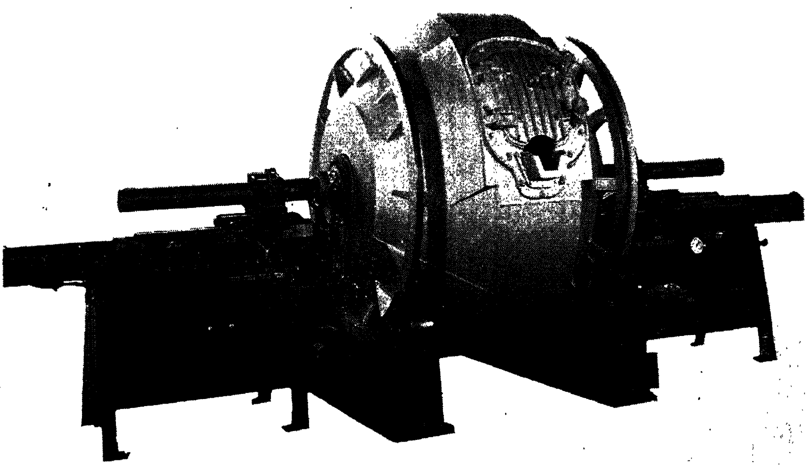


FIG. 50.—Rocking arc furnace. (Courtesy *Kuhlmann Electric Co.*, Detroit Electric Furnace Division.)

(*b*) Change the height of the load to be heated. This possibility will be discussed in the section on the dimensions of the furnace chamber (see below, "Size and Shape").

(*c*) Bring, either continuously or intermittently, fresh parts of the load in contact with the heat source or to points at which they are exposed to it. The rocking arc furnace is built with this purpose in mind (Fig. 50). In this type of furnace, the chamber rocks around a horizontal axis. The two electrodes (single-phase furnaces only!) are in the axis of the furnace. In other designs the roof rotates around a vertical axis, thus bringing the arc in contact with new parts of the load. A similar design inverts the movement. The bottom of the furnace chamber is slowly swung around a vertical axis drawn through the center of the standing roof. These designs, although quite promising in appearance, have not as yet been introduced into the United States. Detailed analyses of the operation and success of these designs have not been published. The same is true of an attempt to rotate the arcs by inductive forces and thus move them over the surface of the steel, while the roof and bottom of the furnace chamber stand still.[28] These latter designs seem to promise considerable improvement in uniformity of the product and considerable savings in burned material.

II. THE FURNACE BODY

In the literature only the inside dimensions of the usually round shell of arc furnaces is mentioned as the characteristic figure. It seems important, however, to distinguish between outside and inside dimensions of the lining. The inside dimensions give the holding capacity, while supplying the outside dimensions affords some idea of the thickness of the lining of the furnace.

A. Design

1. Size and Shape

From previous considerations the bath capacity (holding capacity in tons) can be determined if the density of the load is given. The necessary bath volume can readily be calculated. In some cases, however, this is not identical with the hearth volume: The melting zone may be surrounded by material which is to be preheated and will melt later on; the hearth volume is larger than the volume of the melting zone. For the melting of iron and steel, the hearth volume is fairly well defined.

In steel melting furnaces and foundry furnaces, the density (of the charge) to be introduced into the calculation must be selected. In furnaces charged with liquid metal, the density may be taken from any handbook. Difficulties arise in estimating the average density of the charge if loaded in a solid state, because the apparent density depends

[28] G. E. Evreinoff and S. Y. Telny, *Rev. mét.*, **24**, 57 (1927). M. Suttor, *Bull. Sci. Assoc. Ing. Elec. Sortis Inst. Electrotech. Montefiore Liège*, **18**, 165 (1938).

on the size of the individual pieces. The following figures (taken from Stansel [29]) may be considered indicative:

Kind of scrap	Apparent density; lb/cu ft
Automobile scrap	42
Light railroad scrap	45
Compressed bundles	50
Crushed turnings	56
Heavy railroad scrap	85–90
Rails	132
Foundry gates and risers	135
Billet ends	180–190
Short heavy crop ends	240–250
Large scrap	300
Steel (for the sake of comparison)	480

The figures thereby obtained give only the hearth volume, which is naturally always bigger than the bath volume. The roof must be kept at a distance from the arc in order to prevent overheating through the arc. This will be discussed in greater detail in connection with the roof.

Once the volume of a furnace is determined, the shape must be selected. Are melting furnaces—except for very large furnaces calling for more than three electrodes—are usually cylindrical, though sometimes conical. The horizontal cross section, however, will almost always be a circle. With the (three) electrodes equally spaced about a circle, optimum temperature uniformity is obtained. (Only exceedingly large furnaces and those utilizing square electrodes as in ferro-alloy production sometimes have another shape, either of elliptical or of rectangular cross section.) The conical shape was introduced for several reasons: This type of furnace has a large surface area per unit of bath volume and hence a large area for receiving reradiation from the roof; the rate of melting is increased and the power consumption lowered; and further savings are made through the use of thicker lining in the corners. The disadvantage of a furnace with the conical shape as compared with a furnace with straight side walls and the same outside shell lies in the decreased holding capacity. For steel which needs an extended period of refining the conical shape is disadvantageous because the loss of volume causes so much higher power consumption during the refining period that the savings through the quicker melting period are outweighed. For processes involving little or no refining, the savings through rapid melting are decisive and the conical shape is preferred.

In steel furnaces of cylindrical shape, diameter and height are next determined. Selection of the diameter : height ratio depends upon several factors, which are discussed in the following four sections.

[29] N. R. Stansel, *Industrial Electric Heating*. Wiley, New York, 1933. Figures corrected on the basis of the more recent publication of T. J. Ess, *Iron Steel Engr.*, **21**, AF7 (1944).

(a) Heat Losses per Unit Time

If the thermal resistance of the lining were uniform on all sides, an equilateral cylinder would provide the smallest losses because its surface area is smaller than that of any other cylinder of equal volume. But, as explained below, the thermal resistance is not uniform, the resistance of the roof being considerably poorer than that of the sides or the bottom. The losses per unit of inside area through the roof therefore are very much larger than those through the side or bottom. To offset these greater losses, it is desirable to keep the diameter of the furnace to a minimum. While, theoretically, there is of course a limit of decrease of diameter even from the viewpoint of heat losses, in practice this limit will never be reached. The selection of a larger diameter (or better, diameter : height ratio) than would be desirable from the viewpoint of heat losses per unit of time is dictated by the following three factors, (*b*), (*c*), and (*d*).

(b) Spacing of Electrodes

The electrodes set a lower limit to the diameter : height ratio for both mechanical and thermal reasons. Not only have the electrodes considerable thickness, but they are held in the roof by water-cooled rings which need additional space. Roughly, it may be stated that the rings have an outside diameter twice that of the electrodes.

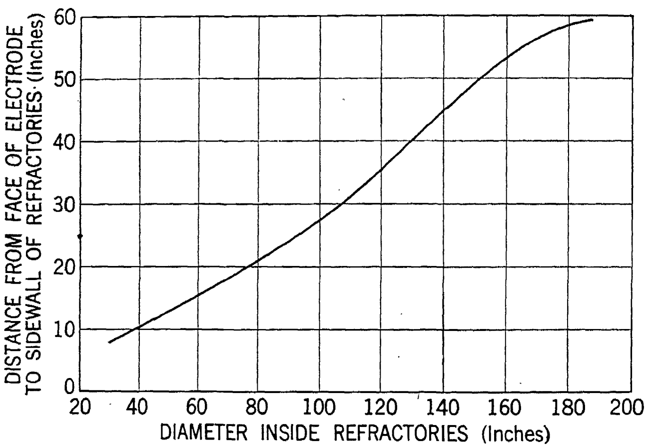


FIG. 51.—Distance of electrodes from sidewall.

Two thermal reasons for distant spacing of the electrodes should be mentioned here: the danger of mutual heating of those portions of the electrodes which are above the cover; and the danger of overheating the charge between the lower tips of the electrodes because of the high concentration of heat in this region. The electrodes should be kept a certain

distance from the walls. Figure 51 shows this average distance as a function of furnace size. Figure 52 shows the diameter of a circle drawn through the center of the electrodes *vs.* the electrode diameter.

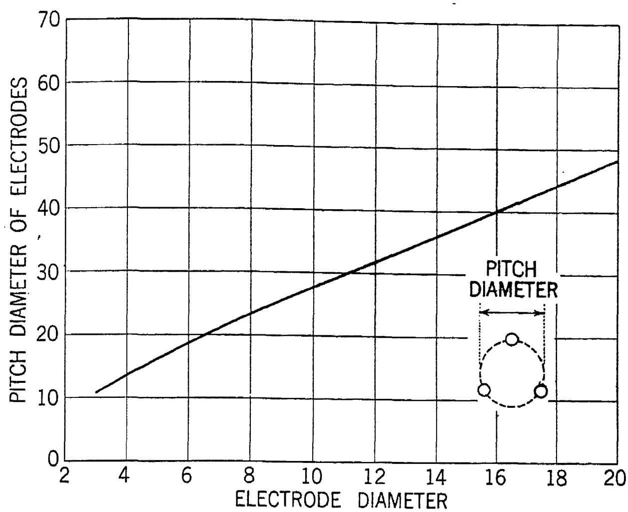


FIG. 52.—Spacing of electrodes.

(c) *Speed and Uniformity of Melting*

The diameter and the diameter : height ratio of the furnace chamber determine the area of the charge exposed directly to the arc; the remaining surface receives only that heat which reradiates from the roof. If the diameter is too large as compared with the depth, the electrodes will be very far apart. The part under direct action of the arc melts more rapidly than the other parts. Following the layout, there would be either unmolten scrap piles remaining on the sides (close to the sidewalls) or in the center between the electrodes. There is an apparent discrepancy between this statement and that made in connection with cone-shaped furnaces. The latter have been limited so far to furnace capacities of 35 tons. For such capacities, the difference in these statements can be readily explained by the fact that, in the conical furnace, temperature differences at lower levels are easily equalized by the reduced diameter. A limitation to the diameter : height ratio is given by the danger of too big a temperature difference between top and bottom, with the consequent possibility that the bottom part of the bath may freeze to the lining. This danger is especially grave if fresh material is loaded frequently during the melting period. The cold material sinks to the bottom and tends to become the nucleus of a freezing zone.

(d) *Metallurgical Considerations*

In any process in which chemical reactions between bath and slag occur, a high depth of bath causes a longer reaction time. While the parts of the bath near the slag react quickly, the parts further away must react by diffusion. Metallurgical reasons call for moderate bath depths, especially where extensive refining processes are involved.

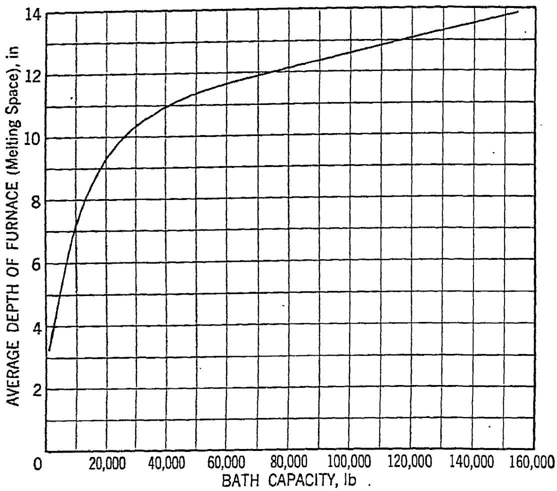


FIG. 53.—Average depth of furnace *vs.* bath capacity.

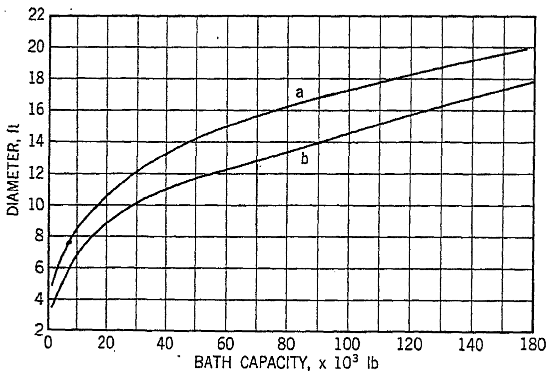


FIG. 54.—Diameter *vs.* bath capacity. Curve *a*, inside shell diameter; curve *b*, inside diameter of lining.

From all these considerations, industry has adopted dimensions approximately as shown in Figure 53 and in Figure 54, Curve *b*, the former showing the average depth of the melting chamber, the latter the diameter of the melting chamber, both curves being plotted *vs.* furnace

capacity. The preceding points regarding the diameter:height ratio are summarized in Table IX.

TABLE IX

FACTORS INVOLVED IN SELECTION OF DIAMETER: HEIGHT RATIO

For deep baths (small diameter)	For shallow baths (large diameter)
Roof problem	Space for electrodes
Uniformity of melt	Reaction area
	Speed of melting

2. Roof

While the sides and bottom of the arc furnace are protected against excessively high temperatures by the charge and the bath itself, the roof is exposed to the high temperature of the arc. If the roof were covered on the outside by a perfect insulator, the inside surface would reach the same temperature as the arc. There is no refractory material available which can withstand such temperatures for a reasonable length of time. The actual life of the lining of furnace roofs depends on a large number of individual items which can be conveniently classified into three groups: temperature level; duration of exposure to high temperature; and thermal expansion of the lining material.

(a) *Temperature Level*

While the temperature of the arc is the same in all furnaces, the greater part of the roof is exposed to the bath only. The higher the operating temperature of the bath, the lower is the life expectancy of the roof. In foundry furnaces, for example, small castings are usually cast at higher temperatures than are larger castings. Furnaces, when used for smaller castings, will therefore have a shorter roof life than when used for large castings. Similarly, a large proportion of scrap of high melting temperature in the charge will result in shorter life of the roof. Even with the same bath temperature, the temperature of the lining may change to a considerable extent. The most important cause of different temperatures of the lining for the same bath temperature is the distance of the bath level from the lining. Figure 55 illustrates the length of life of the roof (expressed in number of melts *vs.* distance from the arc or bath to the roof). The figure refers to silica roofs of rather small European furnaces.[30] The nature of the slag and its thickness influence the differences of temperature between bath and roof.[31] Also, the type of steel has its effect on roof life. High-manganese steel is melted with long

[30] S. Kriz, *Stahl u. Eisen*, 49, 417 (1929); abstracted in *Iron Age*, 124, 461 (1929).

[31] C. A. Brashares, *Trans. Am. Foundrymen's Assoc.*, 49, 1010 (1941).

arcs and therefore results in a comparatively short roof life. It is interesting to compare Figure 56, which indicates the height of the roof (above sill level) *vs.* bath capacity. This figure refers to the practice in the United States.

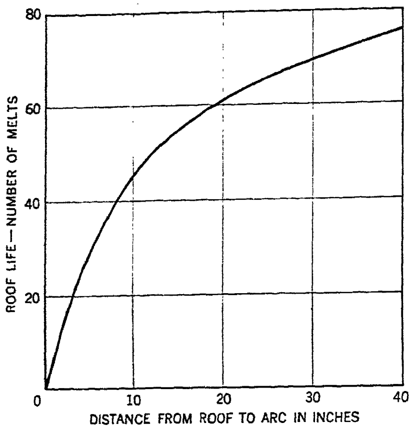


FIG. 55.—Roof life *vs.* distance from arc to roof (after S. Kriz).

The temperature level is important in connection with roof life, because the refractories are operating close to the failing point. In this connection silica brick has two important advantages over other materials: higher strength up to near its failing point even though its refractoriness may be lower than that of other materials; and loss of strength through impurities to a lesser degree than other materials.

By way of example, it is noted that silica brick can stand a load of 25 lb per sq in at a temperature of 2900 F, whereas high-heat-duty fireclay brick when exposed to the same load may undergo a deformation as high as 6% at a temperature of only 2462 F (ASTM requirements).

Oxides of lime and iron (and others) may be contained in furnace dust. The furnace lining may eventually become impregnated with such oxides, and silica brick under such conditions maintains more of its strength than does fireclay brick, even of the high-duty type. Silica bricks have been improved in recent years, for example, by selecting desirable sizes of the quartzite particles used in the manufacture of the bricks.

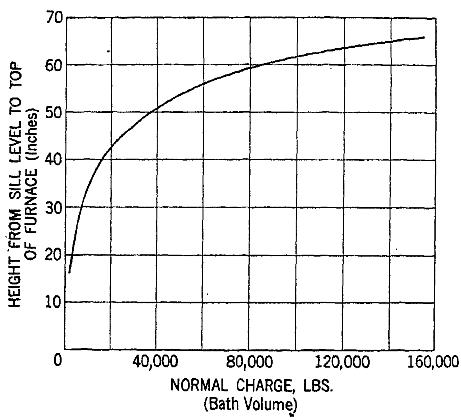


FIG. 56.—Distance from roof to sill level *vs.* bath capacity.

High-alumina bricks are not as yet frequently used in roofs, but their refractoriness is very satisfactory. As far as refractoriness is concerned, fireclay bricks, which are a mixture of alumina and silica oxide, are least desirable. For further details see Brashares.[31]

(b) *Duration of Exposure*

Obviously, the longer a roof is exposed to extreme high temperatures, the shorter its life will be. The biggest factor concerning length of exposure is the duration of the refining period.

(c) *Thermal Expansion*

One of the major causes of roof failure, especially with the most commonly used refractory, silica, is spalling. Spalling is due mainly to differences in thermal expansion between various parts of the bricks. Since thermal expansion varies with temperature, if different parts of the roof have different temperature, spalling will occur easily.

Different temperatures at various parts of the roof may be caused by too rapid melting. Extreme differences result chiefly along the thickness of the roof rather than over the surface. Another factor causing temperature differences is the intervals between melts. If they are sufficiently long, the roof will cool below the critical range (for silica, 200 to 1200 F) and subsequently must be heated up again through this range. The heating-up of the roof is always rapid and therefore shortens the life of the roof, so that long intervals between the melts tend to decrease the life of the roof considerably.

Spalling can be caused not only by sudden temperature changes but also by nonuniform temperature distribution in space. If the charge is unevenly distributed in the furnace, local temperature differences will occur which may well result in different temperatures in different parts of the roof, and consequently in cracking. Temperature differences in the brick and the resulting stresses are a function of the size of the brick: the smaller the size of the brick the smaller the stresses and the better the life of the roof.

In view of all these influences it is clear that figures on the life of roofs must be considered with utmost caution. The following values are presented only for information as to the order of magnitude: Silica roofs in acid furnaces have a life expectancy of from 200 to 300 melts, with an average life expectancy of 250 melts; and silica roofs in basic furnaces have been observed to stand up for between 80 to 200 melts, with an average of perhaps 100 to 150 melts.

The short life of roofs lined with silica has led to a number of investigations to find a material which would be technically and economically superior to silica. On this point American and European practice seem to be contradictory: American authors contend that sillimanite has a longer life than silica, while European authors hold that the life of sillimanite is no longer than that of silica. The high price of sillimanite prevents, for the time being, its use as a lining material for the entire roof except for small units up to approximately one ton, when rammed silli-

manite is used frequently. It has been suggested that sillimanite be used only at danger spots, as around the electrodes, and possibly as a sort of "spine"—a network of sillimanite consisting of 4–6 radial spokes each 9 in wide.[32] For the corners, especially for intermittently operated furnaces, high-alumina bricks are sometimes specified. Occasionally super-duty fireclay bricks are used.[33] Figure 57 illustrates a typical lining of an arc furnace roof.

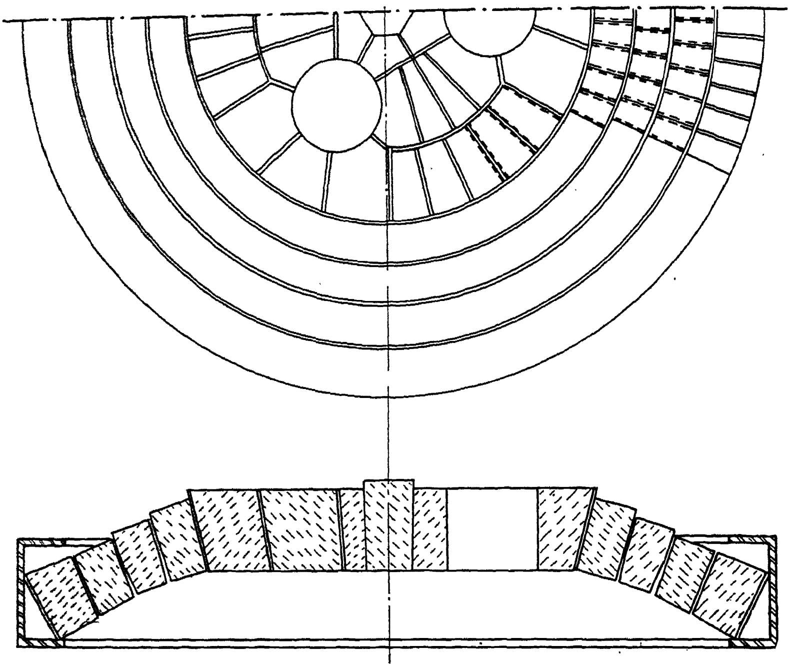


FIG. 57.—Typical cross section through roof. (Courtesy *American Bridge Co.*, Pittsburgh, Pa.)

In modern arc furnaces, the roof is so designed that it can quickly be removed, thus allowing top charging. Special mechanical features which enable a roof to be removed readily are explained below (Figs. 58–62). Top charging consists of removing the roof from the furnace and placing the load suddenly into the melting chamber. The oldest and simplest way of removing the cover was to tilt it out of the horizontal plane (Fig. 58). While this method is very simple, it has the great disadvantage of loosening the bricks and putting increasing stress on

[32] A. V. Laun, *Trans. Am. Foundrymen's Assoc.*, 47, 534 (1939).

[33] "Industrial Survey of Refractory Service Conditions in Electric Furnaces Used in Steel Manufacture" published by the ASTM Committee C-8 on Refractories (August, 1943).

them. Their life is shortened to such an extent as to outweigh the advantage of top charging. This method has therefore been abandoned except for small furnaces of the laboratory type. All modern designs provide for lifting the roof perpendicular to its own plane. During lifting it is essential to prevent any bending of the roof ring in order to avoid additional stress on the lining: One design uses extension rods (Fig. 59); others provide for vertical suspension of the roof at four points. Lifting of the roof is carried out by an hydraulic ram or by electric winches. Once the roof is lifted it must be taken away from over the furnace to permit quick charging. The roof is either swung around, rotating around a ram (Fig. 60), or driven away in a gantry, which also carries the winches (Fig. 61). Figure 62 reproduces a photograph of a furnace with this type of roof, suspended from four chains.

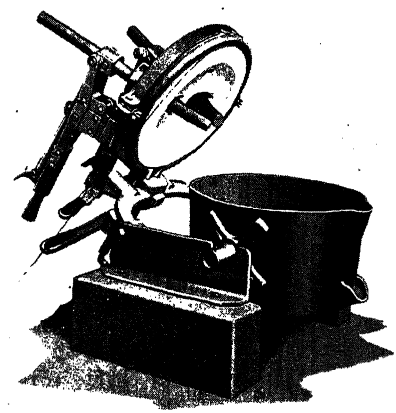


FIG. 58.—Cover tilted out of horizontal plane. (Figures 58–60, courtesy *Pittsburgh Lectromelt Furnace Corp.*, Pittsburgh, Pa.)

3. Sidewalls, Bottom, and Lining

The bottom and sidewalls of the furnace chamber are held together on the outside by a heavily welded or riveted steel shell 0.5–1.25-in thick. Openings which are closed by doors are provided for pouring the metal and drawing slag. The door frames are of structural or cast steel. The sidewalls of the shell are usually either cylindrical or conical, increasing in diameter from the bottom to the top. In one case of a very large furnace having two sets of electrodes, an elliptical shape is used. In round furnaces, the cross section of the bath itself is always conical (that is, the bottom of the bath has a smaller diameter than the top), making it easier to work on the furnace and giving greater durability to the structure. Because of the conical shape of the bath, a conical shape of the shell yields a considerable saving in refractories as compared with cylindrical shells. On the other hand, the lining of the sidewalls and the bottom have so high a life that the saving is almost entirely in the first cost and is not much noticeable in maintenance cost. In first cost, the percentage saving through such a shape is not very high because of the relatively high cost of the electric control and tilting equipment. The bottom of the furnace shell was previously dished in almost all installa-

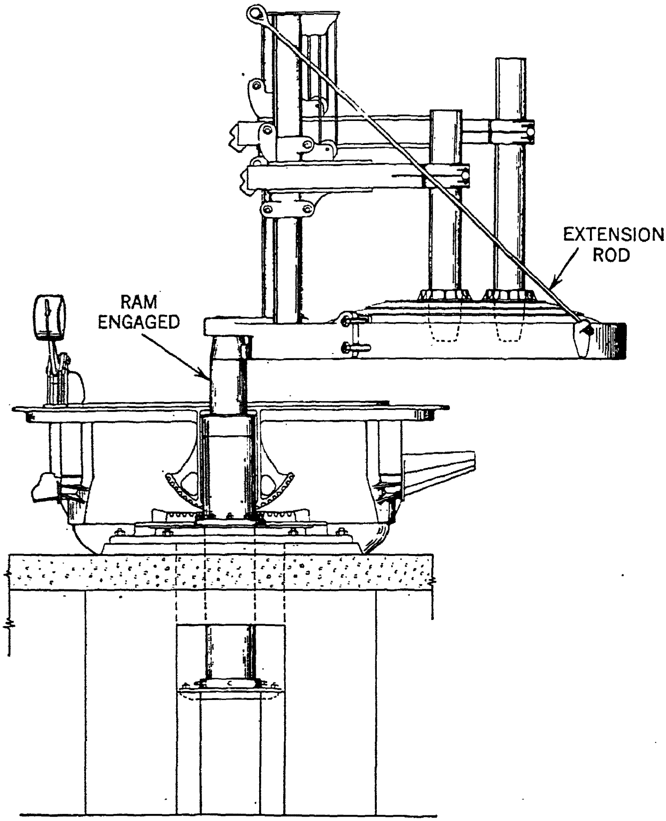


FIG. 59.—Suspension of roof from extension rods.

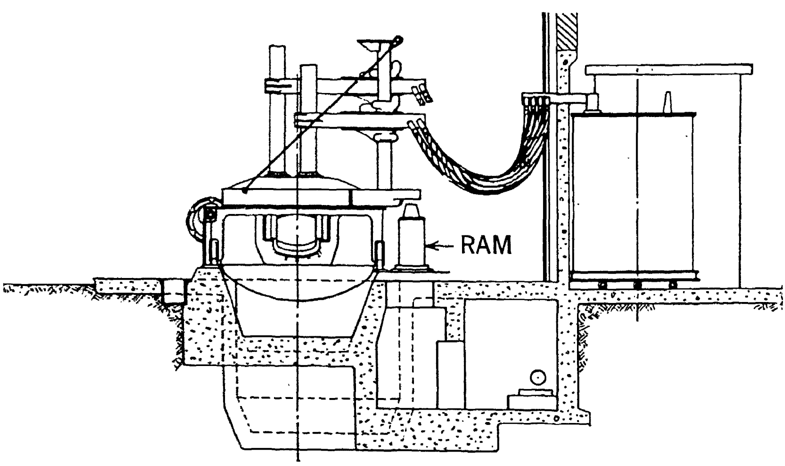


FIG. 60.—Roof swinging around ram.

tions. Now in some instances flat bottoms, which make possible a simpler design of the shell as well as of the refractory lining, also are used. The dished bottom comes closest to the theoretically optimal form—the sphere, which gives smallest surface for a given volume of the inside furnace space. Because the thermally irregular shape of the lining pushes the more effective insulating materials toward the outside surface, where they spread out over a large area, the value of the dished bottoms from a thermal point of view is perhaps somewhat controversial. Both de-

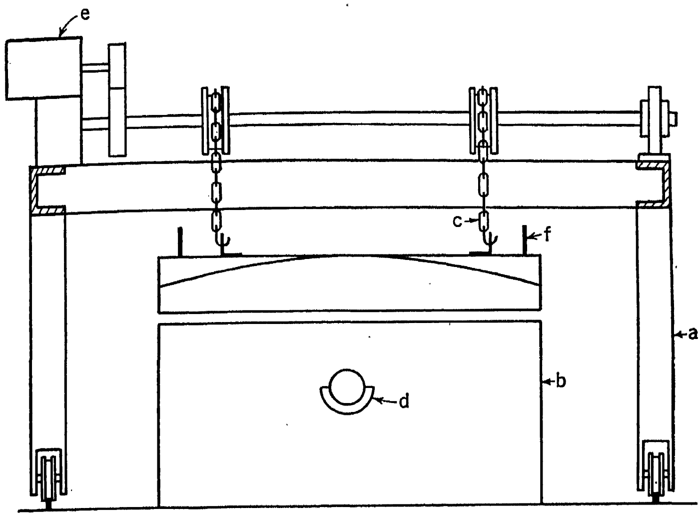


FIG. 61.—Arrangement of gantry-type top-charge furnace: *a*, gantry; *b*, furnace shell; *c*, four suspending chains; *d*, spout; *e*, lift motor; *f*, prongs.

signs—flat bottom as well as dished bottom—can be found at present, even in new furnaces. Certain parts of the shell are water cooled, especially the door frames (see page 115).

It has become customary to correlate the inside shell diameter and the holding capacity of the furnace, a correlation which is not quite logical because of differences in thickness of lining. It would be desirable to indicate the inside dimensions of the melting space along with the shell dimensions. For the sake of completeness, however, Figure 54, Curve *a* shows the shell diameter as function of the holding capacity.

The steel shell is always lined with refractory material. The refractory lining near the shell consists of bricks, while on the inside, near the melting space, rammed-in or burned-in material is used. At and above the slag line, different material is used from that on the bottom and on the lower parts of the sidewalls. Generally speaking, there are two types of lining: basic and acid. Figure 63 shows diagrammatically the lining

of an arc furnace, the left side of which is "acid" and the right, "basic." The acid process is used mostly in steel foundry work, while the basic process is the rule in ingot production. The acid process does not permit the control of phosphorus and sulfur in the melt: the amount of these materials present in the raw materials will appear unchanged in the

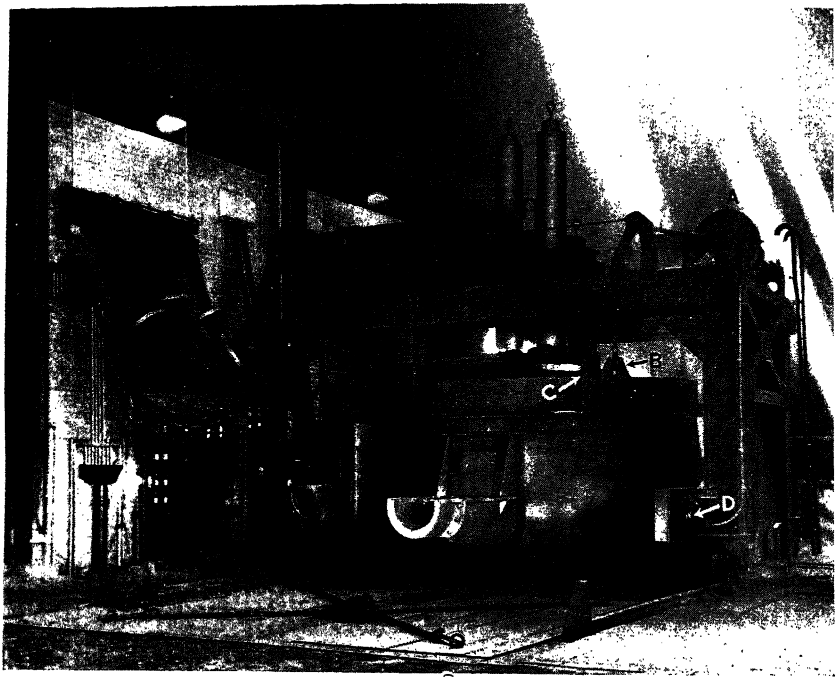


FIG. 62.—Gantry-type removable roof furnace. *A*, roof lift; *B*, suspension of roof; *C*, pins for stiffening roof in lifted position; *D*, gantry travel. (Courtesy *American Bridge Co.*, Pittsburgh, Pa.)

melted product. In the basic process, however, the amount of phosphorus and sulfur can be easily altered through application of appropriate slags. If raw material of correct analysis can be obtained, the acid process is far more economical, the refractories used in the acid process being less expensive than those needed for the basic process.

The melting time—as well as the time for refining—are reduced in the acid furnace. The shorter melting time is due to lower heat losses, leaving more energy free for melting purposes. The shorter time for refining is due to the fact that both the bottom and sidewalls participate in the process of refining to the same extent as does the slag. Consequently, a much larger area of contact is available between steel and reacting material than in the basic furnace. The shorter melting time

naturally means increased output and lower power consumption. Finally, the metal is more fluid and the slag more viscous in the acid furnace as compared with the basic.

The main reason why the acid furnace is not more frequently applied to ingot melting lies in the difficulty in obtaining scrap of guaranteed uniform composition. Acid refractories have a longer life than the basic ones because they are refractory to a higher degree and are less readily

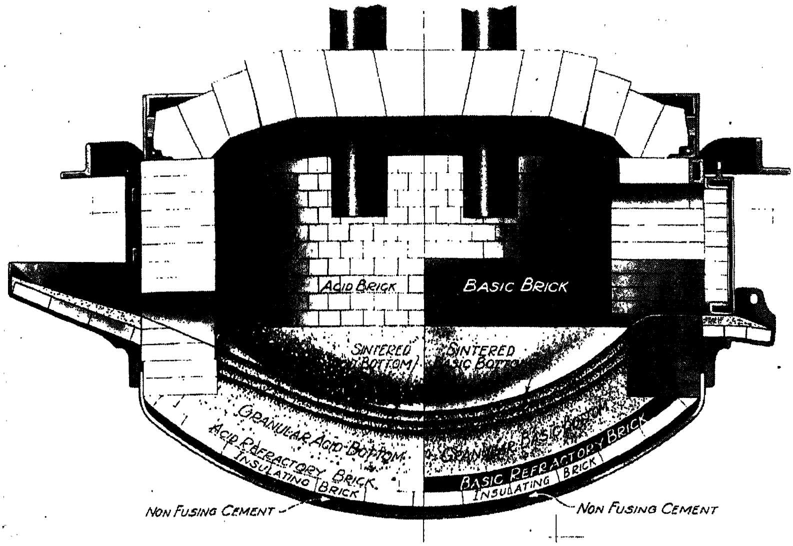


FIG. 63.—Diagrammatic sketch of lining. (Courtesy *Pittsburgh Lectromelt Furnace Corp.*, Pittsburgh, Pa.)

attacked by slags. For this reason, the roof and the sidewalls beyond the slag line are made of acid refractories even in basic furnaces. Acid refractories are silica bricks or metal-enclosed bricks, the latter a type of brick of fairly recent development in which the refractory material is enclosed on five sides by a metal case, the metal case melting and oxidizing when the lining is in place and heated up for the first time. The slag resulting from the case binds very tightly with the brick. A further advantage of the metal-enclosed brick is the protection afforded during shipping and laying of the lining.

Because of the relatively high price of silica brick as well as of magnesite, fireclay is used in corners where temperatures are sufficiently low. The bricks are offset so that the grain material used on the bottom has a better hold.

Rammed bottoms are more refractory than burned-in bottoms. Ramming takes place in successive layers of 4–5-in thickness, rammed

down to some 0.5–3-in thickness. The first heating up after putting in a new rammed bottom should cover a period of about 24 hours. With the tamped bottom, desired contours can be better maintained.

For acid furnaces, ground ganister mix is used and for basic furnaces, grain magnesite. Burning in of bottoms takes place by means of scrap electrodes placed on the prepared bottom. Against these scrap electrodes the arc is drawn. The bottom should be set as tight as possible in order to prevent liquid metal from seeping out. The sides, however, should be built with expansion joints, a length of $\frac{3}{16}$ in per ft having been recommended.[34]

The thickness of the lining is not uniform over the whole cross section. Systematic investigations as to the most desirable thickness have not yet been made. The question of desirable thickness will be discussed in the section on wall losses (pages 106–112). The following data are of general information only: The average thickness of the bottom ranges from 16 in, in very small furnaces, to 20–30 in, in the largest ones; the sidewalls are perhaps 9 in to 2 ft thick, measured at the level of the charging doors.

The life of the sidewalls depends largely on the care and quality of repair: the bottom is repaired after almost every charge, and in some instances the sidewalls also are patched repeatedly. In basic furnaces, the life of the sidewalls may vary from 60 to as much as 400 melts, the average being between 125 and 200 melts. In the acid process, the expected life is between 200 and 300 melts, with an average of 250. The bottom should practically never have to be renewed.

Compared with thermal insulators, refractory material has a high conductivity. The refractory lining is not considered to be an insulation. Early arc furnaces were not insulated; and the refractory material was placed directly against the steel shell. The refractoriness of the lining material was not high enough to withstand elevated temperatures. If elevated temperatures penetrate to greater depths of the lining, the zone increases in which the prevailing temperatures may be too high for the refractoriness of the lining. Now, application of insulation between lining and shell flattens the temperature gradient in the refractory and therefore raises its temperature. While these temperatures may be permissible in many instances, another difficulty arises: If the charge does not shield the arc in all places, the temperature in the lining increases still further in the spot where such shielding is missing.

With improving qualities of refractory and insulating materials, repeated attempts have been made to insulate the sidewalls and bottoms of arc furnaces. The wall losses are the main item in nonproportional heat

[34] J. H. Chivers, *Bull. Am. Ceram. Soc.*, 19, 442 (1940). For more details regarding expansion joints see T. J. Ess, *Iron Steel Engr.*, 21, AF7 (1944).

losses (page 64) and are an important factor in the heat balance. Any savings here will be very noticeable in the power consumption. It would be important therefore to tackle the problem of insulation vigorously and to compare the heat balances of insulated and noninsulated arc furnaces. Unfortunately, no certain data have been made available in the United States. Arnold [35] reports that an insulation thickness between 0.5 and 1.5 in has been tried with inconclusive results. Wallis [36] reports that insulation had been applied for several years to furnaces of 7-ft diameter or more, for the design of which he was responsible. However, sufficient data are not available. It is possible that the application of heat insulation would make more serious the problem of local overheating of the charge under the electrodes (page 77) and that of overheating the refractory of the sidewalls. There are many difficulties but the possible savings are so important that a more systematic study would be worth while. A report by an Austrian user of insulated arc furnaces [37] is inconclusive because of the relatively poor quality of the insulating material available. The report shows a considerable saving in melting time, the melting time decreasing as much as 11%. The insulation was 2.5 in thick, backing up a lining of 14-in thickness. The report is, however, inconsistent in one point: It is stated that the power consumption dropped only 3% because of the insulation. Now, a shortening of melting time can be accomplished only by increased heat supply to the melt. The power input being unchanged, the increased heat supply to the melt can result only from decreased losses. The losses (in kwhr or in Btu) can be considered as the product of the loss per hour (in kw or in Btu per hr) times the melting time. If through the effect of the insulation the rate of heat flow is lowered, the hourly losses decrease. At the same time because of the resulting increase in useful heat, the melting time decreases. The result should be a still stronger reduction of the total heat losses. But the published figures indicate that the melting time dropped with hourly losses unchanged.

4. Doors

Except for very small units, arc furnaces have at least two openings, usually located on opposite sides: an opening (tap hole) in the front for pouring out the molten metal; and a door in the rear (called charging door), which serves mainly to withdraw the slag and to charge the furnace if top charging is not applied. Larger furnaces have one, seldom two, additional openings for stirring, taking samples, etc. ("work doors").

The front opening can be quite small because the high fluidity of the molten product allows a rapid discharge even through a limited area.

[35] S. Arnold, III, *Trans. Am. Inst. Elec. Engrs.*, 52, 839 (1933).

[36] W. B. Wallis, *personal communication.*

[37] H. Weitzer, *Stahl u. Eisen*, 57, 697 (1937).

Generally, a refractory pipe of 4–4.5-in diameter is placed in the lining for this purpose. In order to direct the flow of the metal when it leaves the pipe, a spout is provided. A U-shaped metal frame with a lining of 4.5 in burned magnesite (basic furnaces) or ganister mix (acid furnaces) backed up by fireclay brick serves this purpose (Fig. 64). The length of the spout depends on the arrangement of the pouring pit. As an indi-

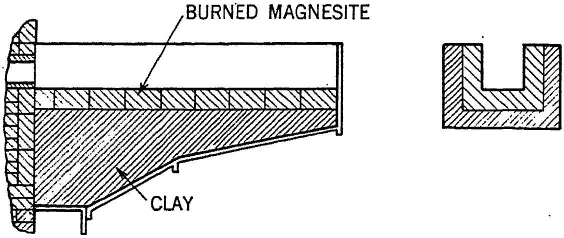


FIG. 64.—Lining of pouring spout.

cation of spout size, it can be said that lengths of 6–6.5 in apply to furnaces of 12-ft shell diameter or more, while smaller furnaces may have a spout of only 5.5–6 in.

In some instances, the rear (slag) door is also provided with a long spout; in most cases, however, a short plate, frequently not even lined with refractory, is used, over which the slag is withdrawn. In that case, a slag apron of steel sheets is applied to protect any part of the moving mechanism against falling slag.

The design of the rear and slag doors is in general the same and can be considered together. Various reasons, explained below, lead to the following general door sizes: The main door opening has a height of approximately 3.5 ft (measured to the top of the arch) and a width ranging from 3.5–5 ft, depending on the furnace size; the side doors are usually smaller, a height of 2.5 ft and a width between 2.5–3 ft being considered as normal.

The doors are usually *lined* with firebrick of 4.5-in thickness. The small thickness obviously increases the heat losses per unit area as compared with the rest of the wall. The door jambs are generally lined with silica brick.

The *door frame* is made of structural or cast steel and is water cooled to prevent warping. The frame is hollow, the water entering at the top and being withdrawn at the bottom. An incline of the inner surface helps to keep the bricks in place. The opening in the sidewall naturally weakens mechanically the refractory lining as well as the shell. The shell is therefore reinforced by a frame of horseshoe shape surrounding the opening. This "horseshoe" is sometimes also hollow and water cooled. Figure 65 shows the arrangement of this type of reinforcement.

A door for an opening 4 ft high and 2.5 ft wide weighs (including a 4.5-in lining) approximately 1000 lb. In order to facilitate movement, doors are balanced by counterweights. In some instances, the movement is effected by electric motors or by hydraulic cylinders.

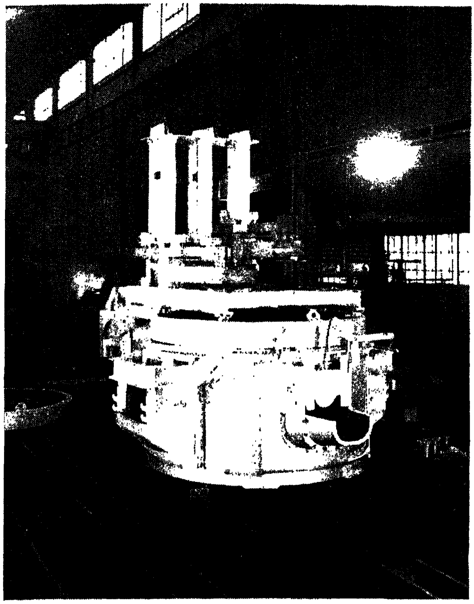


FIG. 65.—Horseshoe-shaped reinforcement of door frame at the left side of the furnace around the door. (Courtesy *Swindell-Dressler Corp.*, Pittsburgh, Pa.)

The doors cause both direct and indirect additional heat losses. The direct increase results from the relatively thin lining of the door and of the heat absorbed by the cooling water of the door frame and the reinforcements of the door jambs. The direct increase is obvious and calls for no further explanation. The indirect increase is caused by a flow of heat from the walls of the opening to neighboring parts of the sidewalls. The indirect increase will be better understood from Figure 66. No figures on either direct or indirect heat losses are available. The losses through the cooling water have been determined in one case in a 7-ton furnace and have been found to be 12 kw. The size of the door in this furnace was 3′2″ wide by 1′1″ high at the sides and 2′ high in the center.[38]

[38] L. Lyche and H. Neuhaus, *Ber. Stahlwerksausschusses Eisenhüttenleute,* **101** (1926); abstracted in *Stahl u. Eisen,* **46**, 780 (1926).

Considerable losses occur while the door is being opened: there is an inrush of cold air and simultaneously an escape of hot gases and air. This loss will be dealt with in the section on heat losses through escaping gases (page 112). Opening the door introduces also an additional loss by

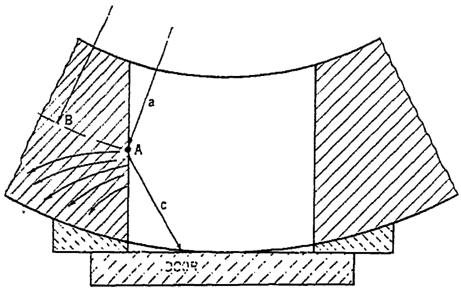


FIG. 66.—Increase of indirect losses through door. Point A receives heat by radiation from arc and bath (a) and reradiates the heat (c) to the door. The temperature at A is higher than at a point of the same radius further in the wall, *e.g.*, B. Therefore heat flows from A towards the outside as indicated by arrows.

radiation. Calculation of this loss can be carried out following a method developed by Hottel and Keller.[39] Figure 12 (page 26) is a graph from which the losses can be computed. It should be noted, however, that their method is based on a nonconductive lining of the door opening. The assumption of a nonconductive lining does not hold; the fact that the lining does conduct heat tends to equalize temperatures, because heat will flow from the hotter parts closer to the inside to the colder (never to the outside surface). Consequently, the parts closer to the inside will show a greater temperature difference against the arc than if the lining were nonconductive, and the total heat loss increases. Kriz[40] mentions a value of 42 kw per sq ft of door opening during the melting period, and a value of 74.5 kw during the refining period. An analysis based on black-body radiation shows that Kriz assumed a temperature of 3240 F during the refining period and a temperature of 2750 F during the melting period. These assumed temperatures depend on the geometry involved. They were computed for a door 4 ft by 2.5 ft, with a 13-in thickness of lining.

There are two ways of cutting down on these loss items: to reduce the opening time of the doors; and to reduce the size of the doors. It should be understood that the figures given refer to open doors. For example, if the door is kept open five times during the melting period,

[39] H. C. Hottel and J. D. Keller, *Trans. Am. Soc. Mech. Engrs., Iron and Steel*, 55, 39 (1933).

[40] S. Kriz, *Arch. Eisenhüttenw.*, 1, 413 (1927).

each time for four minutes, the total time for radiation is 20 minutes. The resulting loss for a door of 10 sq ft would be $20/60 \times 420 = 140$ kwhr. If each opening time is reduced to 3.5 minutes, a saving in the ratio of 3.5/4, or 18 kwhr, would be achieved. A reduction of the size of the door obviously decreases the heat loss.

From the foregoing it may be concluded that the doors should be as small as possible. A small door will result in: greater mechanical strength of the door frame and of the shell; smaller water cooling losses in the door jamb, in the (horseshoe) shaped reinforcement, and in the door frame; and smaller heat losses by conduction in the lining of the door, by conduction in the lining surrounding the door opening (indirect heat losses), by radiation during opening, and by escaping gases. There are two limiting factors determining the possible decrease of door size: At least one door must be big enough to take broken electrode tips out of the furnace, and the larger the door, the easier is the operation of the furnace. In furnaces other than top-charge types, the door must be big enough to permit charging.

No systematic investigations are as yet available from which to determine if the empirically established values for the door opening mentioned above are really as close as possible to the minimum compatible with reasonable ease of operation.

5. Tilting Mechanism

All modern arc furnaces in the iron and steel industry are tiltable. They tilt forward approximately 45 to 50 degrees below the horizontal and, in addition, 10 to 20 degrees backward (for draining off the slag). Furnaces in the chemical industry and ferro-alloy furnaces are frequently not tiltable, but are emptied by tapping.

From the viewpoint of operation, the most desirable tilting mechanism is that which keeps the outer end of the spout in one position (nose tilt). The ladle which receives the molten metal does then not have to be moved while being filled. The second best solution is that which results in a change of position only in a vertical direction of the outer end of the spout. With a small charge, the ladle can still be kept in the same position; and with a large charge, the ladle needs only to be lifted or lowered.

The least desirable design results in a change of position of the outer end of the spout in both a vertical and a horizontal direction. In this case, the operator has to follow the stream of the molten metal with the ladle. Unfortunately, the latter design is the least expensive and easiest to construct. Tilting the furnace over an axis through the center of gravity calls for the least amount of power. Small furnaces up, perhaps,

to a capacity of 6000–8000 lb are built with a trunnion-screw type of tilting mechanism (Fig. 67).

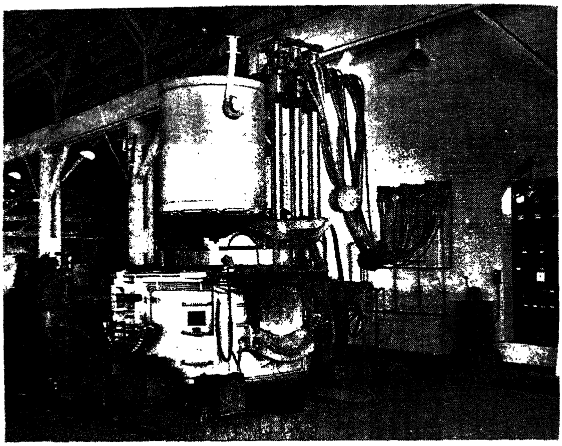


FIG. 67.—Furnace with 1000-lb bath capacity. Trunnion-screw type tilting. The tilting mechanism is motor operated. (Courtesy *Pittsburgh Lectromelt Furnace Corp.*, Pittsburgh, Pa.)

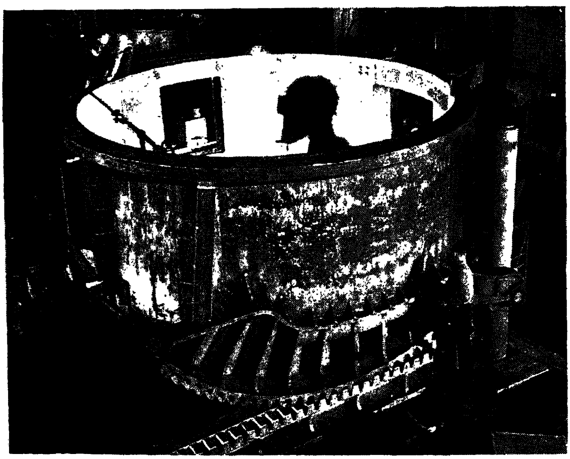


FIG. 68.—Toothed rocker with fixed rack. Furnace under construction. (Courtesy *Pittsburgh Lectromelt Furnace Corp.*, Pittsburgh, Pa.)

Larger sizes are generally built differently. In one design the furnace body rests on a toothed rocker meshing with a fixed rack (Fig. 68). One end of the furnace rocker is then lifted electrically or hydraulically,

resulting in tilting. It is desirable to have a design with positive devices which make it impossible for the furnace rocker to get off the fixed rack. One such means is shown in Figure 69. Tilting is effected through a motor-driven crank. If for some reason or other the motor fails to stop

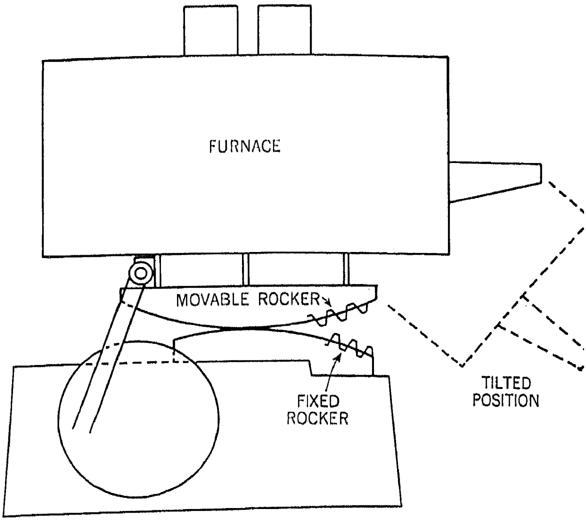


FIG. 69.—Tilting by crank.

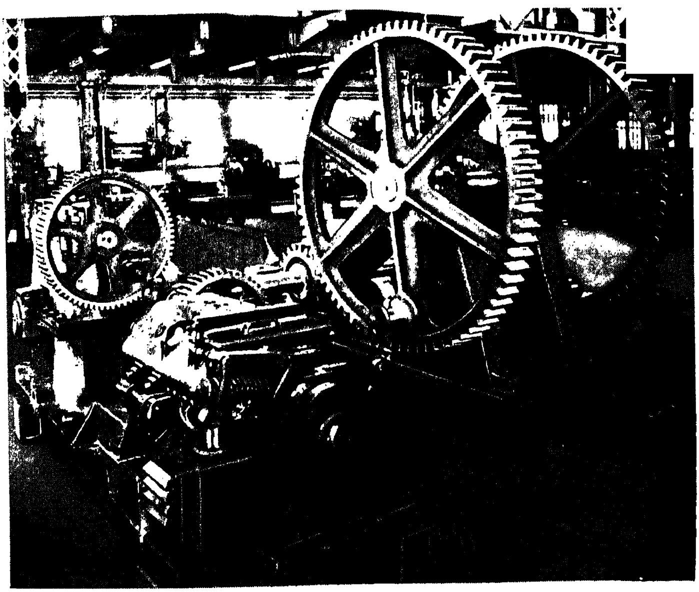


FIG. 70.—Gear assembly for tilting. (Courtesy *American Bridge Co.*, Pittsburgh, Pa.)

nothing happens: the furnace body will continue to swing forward and backward. Another means is by using an hydraulic cylinder: its stroke limits the lift of the furnace. Of course in every case accurate and positive means of stopping are provided, since these are essential in regular operation in order to safeguard good control of pouring. Figure 70 illustrates the gear assembly including the break, and gives an idea of the complexity of the design.

If the outer end of the spout is to be kept in a fixed position, nose tilt must be applied. Figure 71 is a schematic arrangement for nose tilt.

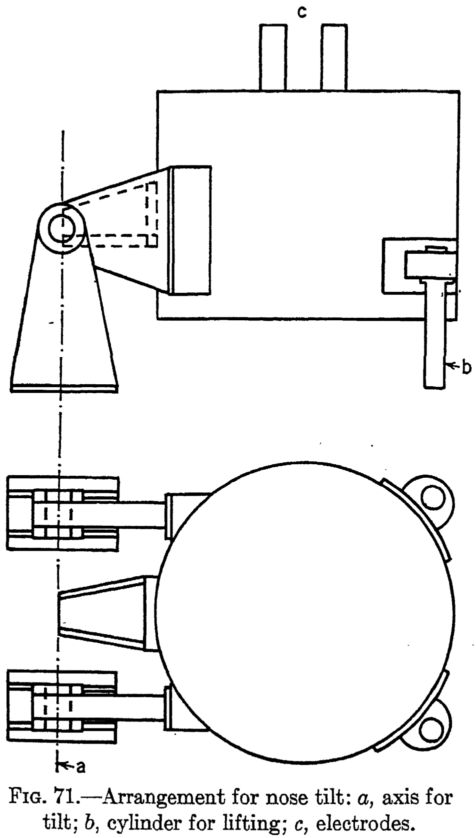


FIG. 71.—Arrangement for nose tilt: *a*, axis for tilt; *b*, cylinder for lifting; *c*, electrodes.

The stress in the furnace shell, as well as the necessary power for lifting, is greater with this design than with either of the other two mentioned above. The tilting motor (mostly low speed, 500–700 rpm, d-c) ranges from a few kw (perhaps 1 to 5) on small furnaces up to a maximum of 20 to 40 on the largest units.

Toothing the rocker prevents the furnace rocker from sliding. Since the toothed rocker must be able to carry the heavy load and still be

accurate, the cost of construction is considerably increased. Therefore a design used in Europe, which avoids toothed rockers and uses a smooth support (Fig. 72), is mentioned here for comparison. Two steel ropes, each anchored on one end and fastened, on the other, to the furnace body, prevent the furnace from being pushed off the base.

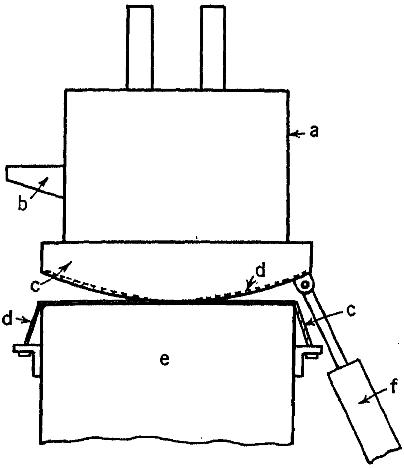


FIG. 72.—Tilting over steel ropes: *a*, furnace body; *b*, pouring spout; *c*, *d*, steel ropes anchored on opposite sides of foundation; *e*, foundation; *f*, tilting cylinder.

6. Charging the Furnace

Rapid charging of the furnace results in cutting down the dead time between two runs. This is desirable from several points of view: First, a larger furnace output is obtained; and second, the heat losses during the loading period are decreased, with consequent savings in the power consumption.

A relatively rapid way of charging, usually applied to smaller furnaces, is by a chute through the slag door. One older way consisted in using large pieces as charge. For approximately the last 15 to 20 years the top-charge furnace has become increasingly popular. In top charging, the whole load is dumped into the furnace after removing the roof (various means of removing the roof were discussed on page 86). The charge is placed in a drop-bottom bucket. One form of the bottom has a number of segments which are tied together by a rope. When the bucket is lowered into the furnace, the rope burns off and the segments drop and discharge the contents of the bucket into the furnace (Fig. 73). Other designs provide for swinging open the bottom by means of levers. Top charging results in a very quick loading of the furnace. If, how-

ever, the load becomes very heavy, then the drop may injure the refractory of the furnace.

In taking the roof away from the furnace, the lining cools very rapidly. This tends to decrease its life. On the other hand, the time

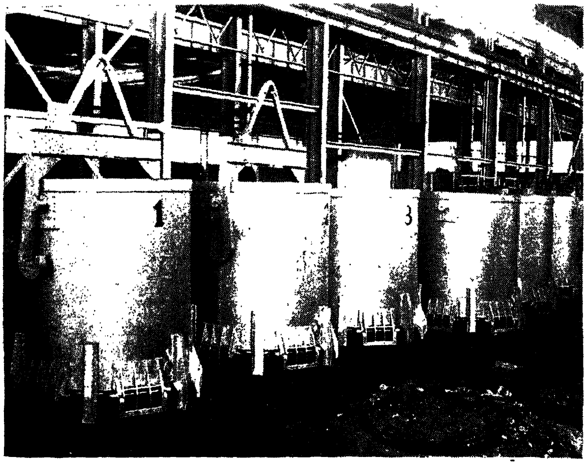


FIG. 73a.—Open charging buckets. (Courtesy *Heltzel Steel Form and Iron Co.*, Warren, Ohio.)

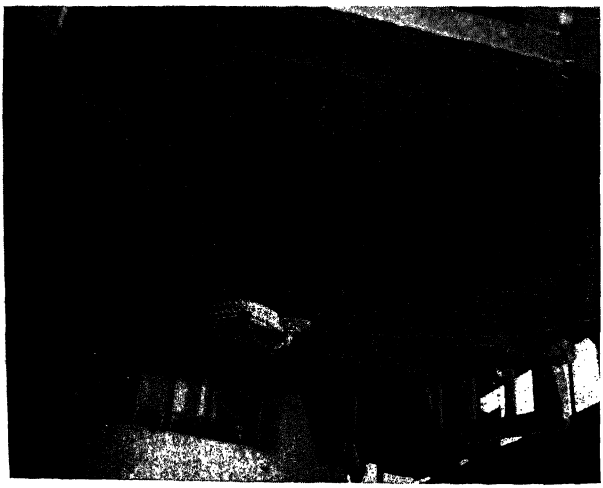


FIG. 73b.—Charging buckets closed with rope tie. (Courtesy *Heltzel Steel Form and Iron Co.*, Warren, Ohio.)

for charging (= cooling) is so short that the temperature drop probably penetrates very little into the lining, less than in furnaces employing the old way of hand charging by chute. Reports published by Pittsburgh

Lectromelt Furnace Corporation seem to indicate that this is true. Two furnaces were compared, both operated at the same time on the same platform, one with a fixed roof and the other of the top-charge type. The refractory cost including labor was $553 for the fixed-roof furnace and only $425 (or 77% of the first value) for the top-charge furnace.

For large furnaces or in plants having several medium sized furnaces a charging machine similar to that used for open-hearth furnaces is very desirable. The roof is not moved and not even temporarily cooled, and yet rapid charging is achieved. The only disadvantage is the necessity for large doors (Fig. 74).

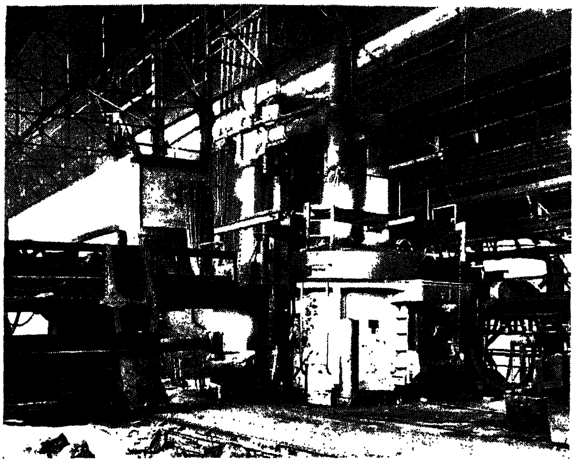


FIG. 74.—Charging machine. (Courtesy *Pittsburgh Lectromelt Furnace Corp.*, Pittsburgh, Pa.)

The advantages of the rapid charging of furnaces can be summarized as follows: lower power cost, because radiation losses from wall and electrodes are decreased (shorter "dead" times); lower electrode consumption (the electrodes oxidize during the "dead" time between the charges and cutting down the "dead" time reduces the burning-off of the electrodes); lower labor cost (because of quicker charging); and higher output. The advantages are not quite so great as would appear at first because part of the "dead" time between two charges is used to repair the lining on sides and bottom. This time cannot be saved by a top-charge procedure.

7. Special Design: Indirect-Arc, Rocking-Type Furnace

Indirect-arc furnaces have been built for many years. The Stassano furnace (Italian) and the Rennerfelt furnace (Swedish) may be mentioned as examples. All indirect-arc furnaces are limited by one big disadvantage: the free-burning arc is very hard on the refractory, especially that

of the roof. The roof is exposed to the arc not only during the refining, but also during the melting period. This difficulty has hindered the widespread use of this type of furnace, so that it is not to be encountered in the United States.

For small units, however, a rocking furnace, designed during the first World War, overcomes the difficulties mentioned above, and provides a very ingenious furnace.[41] This rocking furnace (Fig. 75) has two electrodes arranged horizontally in the axis of the furnace body. The furnace body is either cylindrical or conical, the latter shape being used to improve temperature uniformity. The furnace body is rocked back and forth about this axis, the metal thus continuously changing its position in the furnace and cooling the various parts of the lining. This has a double effect: (1) Every part, with the exception of the door and its immediate surroundings, of the lining is alternately exposed to the arc and cooled by the charge; consequently the lining does not reach excessive temperatures; and (2) the metal can absorb heat more quickly because of the continued contact with a large surface at elevated temperature.

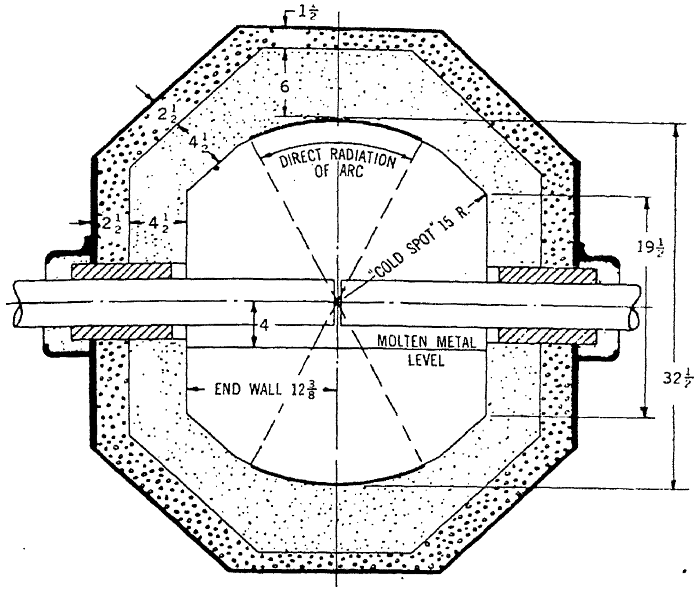
The rocking angle is limited to 180° to avoid spilling out of the metal. Obviously the rocking must be slow while the metal is still solid in order to prevent the bumping of solid lumps too heavily against the walls. A gradual increase in rocking is provided for and a "rocking sequence," once established, can be maintained continuously by automatic control. The furnace body is provided with two large wheels or gears, one on each end: If provided with wheels, each wheel rests on two rolls; if provided with gears, each gear rests on two small gears. The small wheels or gears are driven by motors, which are reversed at regular intervals, thus rocking the furnace body about its axis.

Because of the cooling of the wall by the metal, thinner lining and thermal insulation can be applied, one of the reasons why relatively small furnaces (maximum capacity built to date is 3000 lb cold scrap, 7000 lb molten metal) can compete with large units in regard to power consumption.

Electrode control is provided, the electric system not differing from that for other arc furnaces (see page 136). The principal limiting factor for this type of furnace is its small size, which results in higher labor cost per ton of output as compared with large furnaces. The size of the unit is limited in turn by the electrodes, which are subject to a bending stress that increases with the length of the furnace. In addition, temperature uniformity would suffer by increasing the length of the furnace.

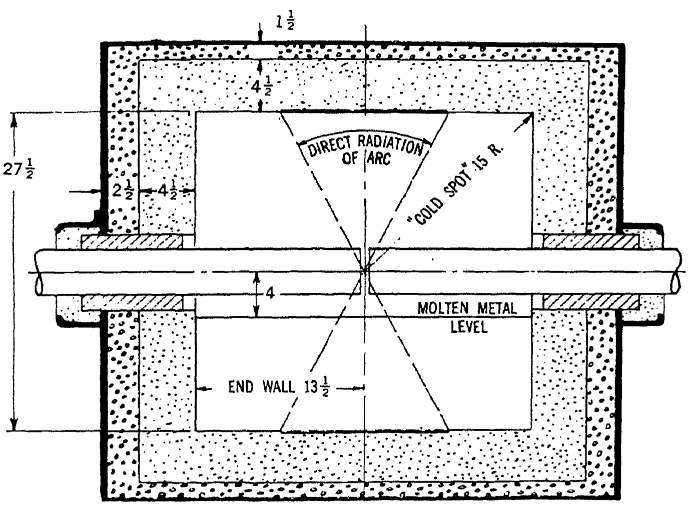
Because of its small size, this type of furnace has been largely limited to foundry work, as far as iron and steel are concerned. In the nonferrous field, it has maintained its place mainly for metals other than

[41] H. W. Gillett and A. E. Rhoads, *Trans. Electrochem. Soc.*, 83, 207 (1943).



CONICAL SHELL

(a)



CYLINDRICAL SHELL

(b)

FIG. 75.—Rocking arc furnace (all dimensions in inches). (Courtesy *Kuhlmann Electric Co.,* Detroit Electric Furnace Division.)

brass. For brass, because of the low vapor point of zinc, the core-type induction furnace has largely taken the place of the indirect-arc furnace. The high temperatures of arc furnaces tend to increase the metal losses, which more than offsets the advantage of the indirect-arc furnace—its capability of intermittent operation.

It is quite interesting to compare the indirect-arc furnace with the radiation type of resistor furnace. The similarity between the two is striking. A critical comparison, along with a discussion of the resistor furnace, will be given in Volume II.

B. Energy Losses

1. Wall Losses

The question of wall losses as well as that of all other losses is highly complex because many of the different types of losses are interrelated. For instance, part of the wall losses may appear under the heading of "losses in cooling water," for certain openings, the rings of electrodes, etc., are frequently watercooled. The portions surrounding the area being watercooled appear to be low in temperature and therefore have, apparently, low losses. But if, for some reason, the watercooling should suddenly stop, the heat losses in these parts would immediately become very high. Calculation of losses is frequently difficult, if not impossible, and systematic measurements of losses have not yet been made. In the United States, no factual breakdown of the total power consumption of arc furnaces has been published. However, Ess (*loc. cit.*, page 72) has given a heat balance which is not based on tests. Publications in Europe [42] usually give such a breakdown for one prevailing condition of service; they do not measure the influence of proportional, nonproportional, and independent losses (see page 64). A wide field of research is therefore still open.

The losses of furnaces involve two different problems: Either one has to deal with the calculation of the losses of a furnace not yet built; or one has to determine the losses of a furnace already in operation, and break down its losses into their various components. The more knowledge gained on furnaces, the more closely will the two ways of approach coincide. At present there is still quite a difference between the losses determined by the two methods of consideration.

An attempt to calculate heat losses in the walls of arc furnaces must be based on the fundamentals explained previously (see page 36). Even though the temperature of the melt, the shape of the furnace chamber,

[42] L. Lyche and H. Neuhaus, *Ber. Stahlwerksausschusses Eisenhüttenleute*, 101 (1926); abstracted in *Stahl u. Eisen*, 46, 780 (1926). E. Mueller, *Stahl u. Eisen*, 59, 126 (1939). S. Kriz, *Arch. Eisenhüttenw.*, 1, 413 (1927). E. Widdel, *Stahl u. Eisen*, 53, 1265 (1933). N. Wark, *Arch. Eisenhüttenw.*, 2, 145 (1928).

the thickness and properties of the lining—all the elements used and mentioned in the basic formula (page 43)—seem to be given, there are still a number of difficulties so serious as to make the calculation of the wall losses troublesome and rather uncertain. Some of these reasons are given in the following paragraphs.

(a) *Nonhomogeneity of the wall.* In order to prevent too high a temperature at certain points of the shell, watercooling is applied locally: door frames and electrode rings are typical places. Bottom, roof, and sides almost always have different types and thicknesses of linings. Local inclusions of different materials, *e. g.*, for reasons of mechanical strength, render calculations still more difficult. Quite frequently, irregular shapes are used for the inside and/or outside of the furnace wall.

(b) *Change of shape and thickness of lining in service.* In a relatively short time of service the thickness of the lining, especially of the lining of the roof, changes. In fact, curves have been established showing the increase of the power consumption with the life of the roof. Such curves will be discussed later (see Fig. 147). Furthermore, not only does the thickness of the lining change, but the inside shape is also subject to change during the lifetime of one lining. In repairing the lining at the slag line, lime and magnesite drop to the bottom; they may change the original shape from the unbroken line in Figure 76 to that shown by the broken line.

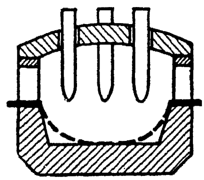


Fig. 76.—Change of shape of hearth. Unbroken line, new lining; broken line, used lining.[42a]

(c) *Heat storage in the furnace walls.* Practically all arc furnaces operate intermittently. Even those running on a 24-hr schedule are, from a purely thermal viewpoint, operated intermittently: after pouring the charge, a cold load is inserted, forcing the furnace temperature to a very low value. During this period, the inside layers of the wall return stored heat to the charge; after melting, the wall again absorbs that heat from the bath. This continual charging and discharging of heat in the furnace wall makes the calculation of heat losses still more difficult.

(d) *Interaction with other heat loss items.* The water cooling mentioned above covers only part of the wall losses. Losses in the electrodes, which obviously must be treated as separate items, influence the wall losses. Escaping gases, the amount and control of which must be calculated from metallurgical considerations, heat the outside of the furnace shell and thus influence the amount of heat flowing through the lining (wall losses).

[42a] F. T. Sisco and S. Kriz, *Das Elektrostahlverfahren.* Springer, Berlin, 1929.

If an attempt were made to calculate the wall losses, the following method might be used, even though only as an approximation:

(1) Divide the inside surface of the wall into elements, each element being backed up by uniform insulation. For example, the whole bottom will probably in many instances form one element. For this element, determine the inside and the outside area, applying Equations (16) and (18a) (geometric mean value). The difficulty in this procedure lies in the determination of the outside area. Reference is made to Figure 77, which shows schematically the lining of an arc furnace. The various layers extend to different depths. A reasonable method of selecting the outside area is shown in the table below.

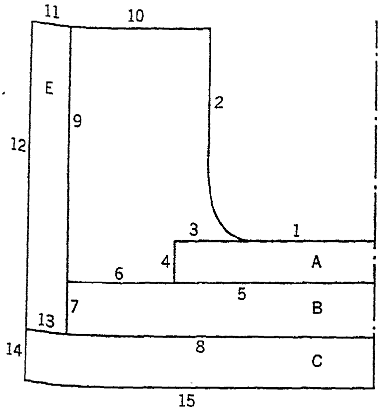


FIG. 77.—Cross-section through half an arc furnace (body lining only). The bottom is composed of three layers, the side of two layers of different materials.

To find the inside and outside areas for heat loss calculation, the following approach is suggested. The wall is considered to be composed of two elements, I (bottom) and II (sidewall), which are mutually independent (see table below). The bottom is composed of three layers, the sides of two. The introduction of area 7 as in the area for layer *C*, element I, is an example of the necessary approximation.

Element	Layer	Inside area	Outside area
I			
	A	1	4 5
	B	4 5	7 8
	C	7 8	14 15
II			
	D	2	9
	E	9	12

(2) Where water cooling is applied, make a separate element. The inside temperature of this element is naturally the same as that of the rest of the furnace chamber. The outside temperature can be estimated approximately as equal to that of the entering water. Thus, with both inside and outside temperatures given, the heat loss follows from Equation (2). By this approximation the difficulties due to the lack of knowledge of the film resistance between the cooling water and the tube surface are avoided.

(3) Where irregularities exist in the lining, as, for example, those due to reinforcing metal extending into the wall or to bricks of different properties locally used for mechanical or metallurgical reasons, make a separate element for this part of the lining without considering any lateral heat flow from this part. This of course may cause a major inaccuracy, as is obvious from the explanations on page 51. But it is the best available approximation, unless elaborate methods of determination are used (*e. g.*, electric analogy method, numerical method, etc.; see page 31). In Figure 78, for example, *A*, *B*, and *C* are materials of different conductivities. Hence there is a heat flow along arrow 1; but since this flow is very difficult to determine, it is neglected, and elements *I*, *II*, and *III* are used instead. All are two-layer elements, *I* and *III* consisting of materials *A* and *C*, and *II* consisting of materials *B* and *C*.

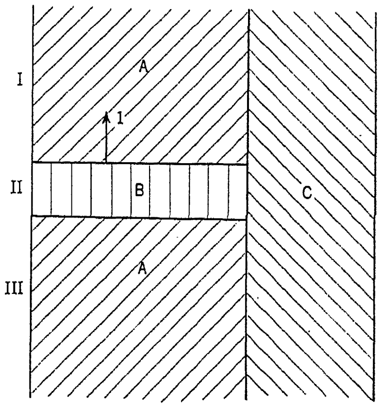


FIG. 78.—Irregularities in the wall. Method of lumping. *A*, *B*, and *C* are the different materials.

(4) Neglect the influence of the electrodes on wall losses when using this approximate approach.

Having discussed the difficulties of the calculation of the wall losses, the actual wall losses in operation will now be considered. Three methods of determination have been used: either the wall losses are considered as "unaccounted balance," including all so-called nonelectrical losses except those by the cooling water; or they are measured by inserting thermocouples into the wall at two different depths. If the thermal conductivity of the wall material is known and if there is no lateral heat flow, the heat loss can be computed from such measurements. The third method consists of measuring surface temperatures and assuming boundary conductance values. Then the heat flow from the surface can be calculated.

None of the three methods is entirely satisfactory. Measurements with a heat-flow meter would indicate local flow conditions; no investigations of wall losses of arc furnaces using heat-flow meters have become known. There is very little information in American literature about the wall losses of arc furnaces; and in European literature, mainly German publications, upon which the following information is based, the data given are of only limited value because of their age—they all date back a considerable length of time and consequently refer mostly to small furnace sizes—and because of the development in this country of insulating (light-weight) refractories, which, if properly used, definitely help decrease the losses. Since these publications are written mostly by steel plant engineers and not by furnace engineers, they tend

to refer all information to output, even when no direct relationship is justifiable. Most energy balances show the heat losses in per cent of total consumption or in kw per ton. But the wall losses are largely independent of the output. They are the most important "nonproportional" heat loss. On the other hand, information about the geometry involved is nearly lacking. For example Kriz[43] giving the wall losses for furnaces in operation, shows shape and dimensions for only a few of the many furnaces discussed, and no information is offered as to the nature of the lining and its thermal properties. Neither the bath temperature nor the steel composition is given which would permit an estimate of temperatures.

The following general conclusions may be drawn from the literature:

(*a*) Losses through sidewalls, bottom and through the roof are to be treated separately. Sidewalls and bottom can be insulated fairly effectively, the former at least up to the "slag line." The roof, however, being exposed to the high temperatures of the arc, cannot be insulated.

(*b*) Since heat losses increase as the lining of any part—side, bottom, or roof—wears out, the wall losses are lower with a new than with an old lining. Because of the more severe wear of the roof as compared with sides (and bottom), the increase of losses from the roof is, on a percentage basis, higher than that of the sides and bottom.

(*c*) Obviously the wall losses are higher for larger furnaces. Figure 147 (page 206) shows the total wall losses plotted *vs.* furnace size. The chart contains two curves, one showing the losses for a new lining and one for an old lining. The curves in the figure represent only the wall losses proper. Since the extent to which water cooling is applied to different parts may vary widely from furnace to furnace, the value of the curves is further limited.

(*d*) The increase of wall losses with life of the lining has been investigated several times. For a 15-ton furnace, Klinar, Reinhold, and Wark [44] indicate for a new lining total wall losses of 240 kw and for a used lining, 395 kw. In this furnace, the increase in power consumption with the life is due principally to the increase of losses in the roof. The latter start (new roof) with 140 kw and increase to almost 240 kw toward the end of the cover life. Interesting results obtained by Kriz and Kral [45] are shown in Figure 79. The life of the roof is obviously very much shorter than that of the sidewall lining. The age of the sidewall lining is given in "number of melts," and is plotted *vs.* power consumption in kwhr per ton. The curves show a zigzag character: As soon as a new roof is put on, the power consumption drops from the previous high value; during the first five melts, the roof (and

[43] S. Kriz, *Arch. Eisenhüttenw.*, 1, 413 (1927).
[44] H. Klinar, O. Reinhold, and N. Wark, *Arch. Eisenhüttenw.*, 2, 151 (1928).
[45] S. Kriz and H. Kral, *Stahl u. Eisen*, 50, 221 (1930).

the sidewalls, which in this case were always repaired when a new roof was put on) is still wet and dries out only gradually, during which period the heat losses drop; after the fifth melt, the wall is entirely dry and the heat losses and power consumption reach a minimum. Then the wear of the lining of the roof starts to show. The power consumption increases

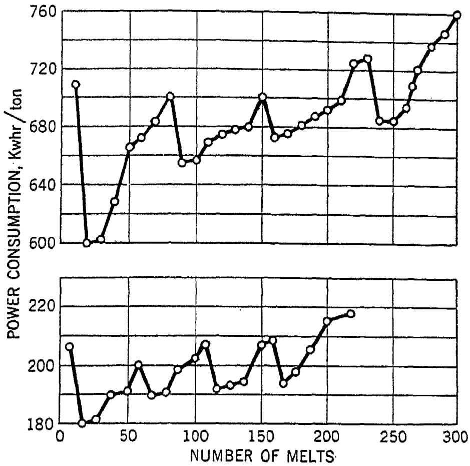


FIG. 79.—Power consumption *vs.* life of lining. Furnace charged with solids (upper graph), furnace charged with liquid (duplexing) (lower graph).[45]

until it finally reaches a point at which the roof must be renewed. In the discussion of this paper, Matuschka mentions another furnace in which the lining was not repaired at each change of roof; during the life of the lining of this furnace, the power consumption of the furnace gradually increased from 600 to 760 kwhr per ton.

(*e*) If an energy balance is made for regularly operated arc furnaces, and if this balance is limited to the melting time (excluding the refining period), results are obtained which at first appear somewhat strange. The total heat expense (losses of the furnace as found from individual measurements plus the useful heat put into the steel) are larger than the "heat income"—electric current plus the heat content of the cold charge plus possible chemical reactions. This situation can be readily explained. The difference disappears if the heat balance is made for the total time of melting plus refining, and if, as "heat income," the heat stored in the lining at the beginning of the charge is also listed and, as "heat expense," the heat content of the lining at the end of the charge. Because of the heat storage capacity of the wall, the time–temperature curve of the outside surface (where the heat losses can be measured) lags

considerably behind the time–temperature curve of the inner surface of the lining. In other words, during the melting period, the charge receives heat from the furnace lining: heat flows from the inside surface of the lining towards the charge. During the refining period, there is a heat flow from the charge into the furnace lining; the lining is brought back to its old heat content. In one specific case,[46] the change of heat content during melting and refining was determined, and was found to be of the order of magnitude of 350 to 550 kwhr (seven-ton furnace).

No quantitative studies are available showing the influence of different lining materials. It follows from general considerations that the lag will be greater for walls of higher thermal diffusivity (page 55) and greater thickness. A large lag in the temperature wave probably means quicker melting. Instead of the charge's receiving heat only from the top (that is, from the arc), as would be the case if there were no heat storage in the wall, the whole wall acts at the beginning of the melting period as a heating surface, resulting in shorter heating time. During the refining period, when this, so to say, borrowed heat must be returned to the wall, ample energy is available. Hence it can be said that such storage of heat is generally desirable. In cases in which the furnace is frequently shut down or does not operate continuously for 24 hours, however, the conditions are different: The heat stored in the walls will be dissipated from the outside surface of the furnace wall during shutdown periods and therefore will be lost instead of being reutilized in a later melting period. In such cases, a lining with as small a heat storage capacity as possible should be used.

A more systematic investigation of these problems would be very worth while.

2. Losses through Escaping Gases

High temperatures in the melting chamber, in combination with the chemical reactions taking place there, tend to produce pressure in the melting chamber, a fact which has hardly received attention. The furnace chamber can never be absolutely tight. Hot gases, which carry away sensible heat, escape around the electrodes and the doors.[47] When the furnace door is opened for stirring, sampling, etc., cold air is drawn into the furnace at the same time as the hot gases escape. Besides this sensible heat loss, additional losses occur because the escaping gases contain carbon monoxide. If this gas were burned within the furnace, additional energy would be liberated, so that there is therefore a loss through the escape of chemically bound energy from the furnace with the gases.

[46] N. Wark, *Arch. Eisenhüttenw.*, 2, 145 (1928).

[47] S. Arnold, in a personal communication, estimates that the pressure in the known chamber usually is below one inch of water.

The composition of the gases and hence the amount of heat lost are different for each process. In addition, the composition changes within each process for the different time elements of the melt. For example, entirely different compositions of gases prevail during the melting and refining of steel. Lyche and Neuhaus [48] report the composition of the gases for a six-ton-2000-kva furnace (Table X). In small steel melting furnaces, the loss through escaping gases amounts to approximately 30–60 kw, which corresponds to about 3–5% of the total energy supplied.

Losses of gases through the door openings can be reduced only by cutting down the opening time to a minimum, but they cannot be eliminated. It is important to make all openings as tight as possible: doors should be tight in the closed position; elimination of warpage helps and is one reason for water cooling of the doors; the electrode rings should fit tight. Italian designers have gone much further in providing nonleaking rings than is customary elsewhere.

A very complicated design is shown in Figure 80. Three water-cooled tubes are provided, *a*, *d*, and *i*. Tube *a* is permanently connected with the roof. Tube *i* is in rigid connection with electrode holders *p*. Spacers (b_1, b_2, b_3, and b_4) between the electrode and tube *a* provide gas expansion chambers (c_1, c_2, and c_3). Tube *a*, sliding in tube *d*, gives a telescopic action which allows for lifting and lowering of the electrode, keeping the

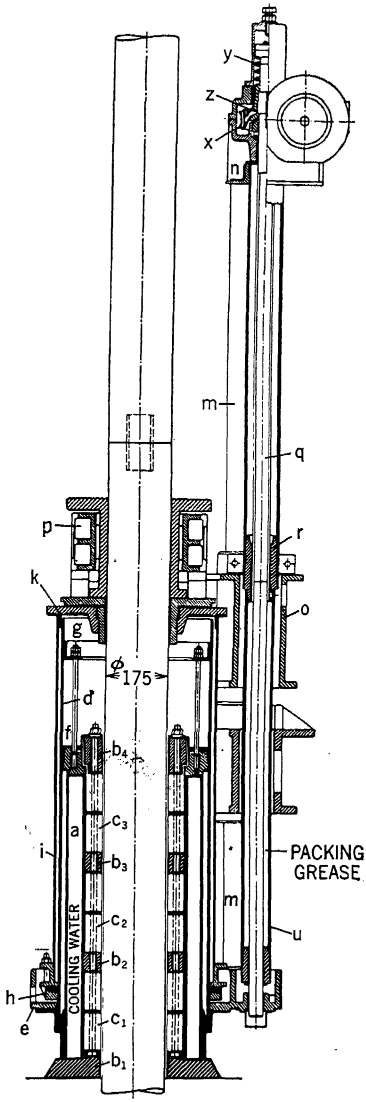


FIG. 80.—Electrode ring (with special protection against escaping gases).[48a]

[48] L. Lyche and H. Neuhaus, *Ber. Stahlwerksausschusses Eisenhüttenleute*, **101** (1926); abstracted in *Stahl u. Eisen*, **46**, 780 (1926).

[48a] R. Taussig, *Elektrische Schmelzöfen*. Springer, Berlin, 1933.

TABLE X

COMPOSITION OF GASES IN A SIX-TON–2000 KVA FURNACE

Composition during	CO_2	O_2	CO	CH_4	H_2	N_2	Not accounted for
Melting period, %	9.96	1.3	15.8	0.24	1.85	70.8	0.05
Refining period, %	2.73	0.25	42.7	—	9.7	44.62	—

latter always separated from the atmosphere. Close fits at *f* and *h* prevent escape of the gases.

Between this very complicated design, which would not be acceptable in this country, and the very simple design of Figure 81, which, however, provides little protection against escaping gases, designs of various degree of complication have been tried. Figure 82 shows a simple shape

FIG. 81.—Welded, water-cooled electrode ring. (Courtesy *American Bridge Co.*, Pittsburgh, Pa.)

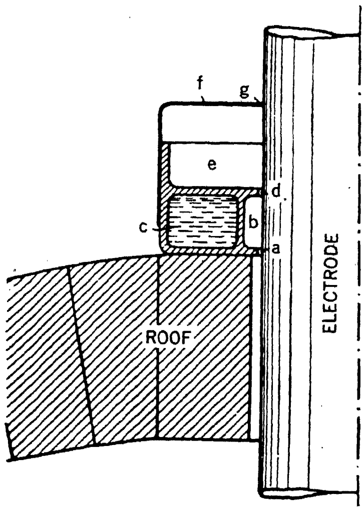


FIG. 82.—Electrode ring with gas chamber: *a*, gas escape from the furnace; *b*, first expansion chamber; *c*, water-cooling chamber; *d*, second annulus; *e*, second expansion chamber; *f*, sheet metal cover; *g*, gas escape.[42a]

which offers some safety against leakage of gases: the electrode ring is provided with an inclined upper surface on which fireclay bricks are laid to keep the gases in the furnace.

In electrochemical furnaces and ferro-alloy furnaces which operate without cover, the amount of these losses is much higher, as in furnaces

for the production of calcium carbide, in which the amount of heat lost in the waste gases (chemically bound, latent heat, heat content of dust) amounts to approximately 35% of the energy supplied by the electric current.

Attempts to utilize the waste gases have been made but have not been very successful. Waste gas utilization in electric furnaces is rarely employed. Design and operation become so complicated that possible savings in heat are offset.

3. Losses through Water Cooling

Water cooling is used in electric furnaces to maintain the temperature in the neighborhood of the cooled part below specified danger levels. For the various parts subjected to water cooling, danger may arise at different temperatures. In arc furnaces, water cooling, besides being applied to the electrical equipment—transformer, impedance coil—is applied mainly to the following parts: the openings in the roof through which the electrodes are brought out (electrode rings); the roof ring, which holds the lining of the roof; the door frame, including reinforcement (horseshoe-shaped, see page 94); and the electric busses. Water cooling of the busses will be discussed later. Water cooling of the body of the furnace will be treated at this point.

The purpose of water cooling the electrode rings is twofold: to protect the lining of the roof; and so to cool the gases that, in escaping, they do not burn the upper parts of the electrodes. The chambers or rings should not fit tightly on the electrodes because of slight unavoidable side movements of the electrodes. Figure 81 shows a typical picture of such an electrode ring. The rings can be welded rather than cast, thus decreasing the weight to be carried by the roof. The amount of water passing through these rings should be so adjusted as to give the smallest permissible amount of cooling. The lower the temperature at the cooled part, the longer will be its life. On the other hand, low temperatures mean increased heat losses. Therefore in each individual case systematic investigations should be carried out to obtain data on the increase of life and the increase of power cost with better cooling. By adding these two cost elements the optimum amount of cooling giving the lowest over-all cost can be determined. Practically the only data available are in the publication of Lyche and Neuhaus,[48] who report on a six-ton arc furnace with Söderberg electrodes. The electrode rings surrounding the electrodes (the diameters of which are not given) cause an average loss by cooling water of 18 kw each over the total melting and refining time of 7.45 hours.

The connection between the flexible leads and the electrodes is also water cooled. In smaller furnaces, the losses at this place amount to about 10 kw per electrode.

The total amount of water used in an arc furnace is quite considerable and ranges from 100 to 1500 gallons per hour, including the water necessary for the transformer. Of course the amount of water depends on the size of the furnace. For steel melting, Figure 83 indicates the average amount of water per pound of furnace output plotted *vs.* furnace

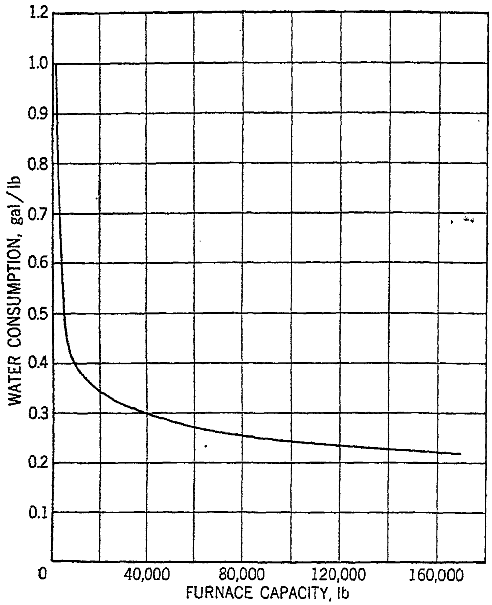


FIG. 83.—Cooling water consumption *vs.* furnace capacity.

capacity. The figures are sometimes much larger in electrochemical furnaces. Wotschke [49] mentions a consumption of cooling water of about 3.4 gallons per pound in the manufacture of calcium carbide.

The amount of water used depends in part on the maximum permissible temperature at any given point that is being cooled. The increase in water temperature in the transformer should not be more than 5–10 C (9–18 F) and that in the furnace parts, not more than 10–20 C (18–36 F). Hence, in some instances, water is made to flow—in series connection—first through the electrical parts and then through the cooled parts of the furnace.

Many furnace operators are tempted to waste the cooling water. The water is turned open all the way, with resulting waste both of water and of heat (power). Application of automatic control adjusting the

[49] J. Wotschke, *Grundlagen des elektrischen Schmelzofens.* Knapp, Halle, 1933, p. 466.

amount of water according to its temperature is feasible and could result in savings. So far it has been considered to be too complicated.

In designing the water-cooling system for the electrical parts, it must not be forgotten that water itself is a conductor. Since water flowing to the ground would result in grounding that part of the circuit, the cooling water is usually not in direct contact with the cooled parts of the circuits. For example, the electrode rings, not the electrodes, are cooled.

Special precaution must obviously be taken to provide water-tight tubing. Water leaking into the furnace, especially under the level of the bath, would evaporate so suddenly as to cause a major explosion.

III. THE ELECTRODES

Electrodes are needed to bring electric energy to that part of the furnace in which the electric energy is transformed into heat. The material from which electrodes are constructed should have the following properties: high electric conductivity (with low conductivity the electrodes would heat up and act as a resistor); refractoriness and high resistance against the atmosphere in the furnace (the electrodes should

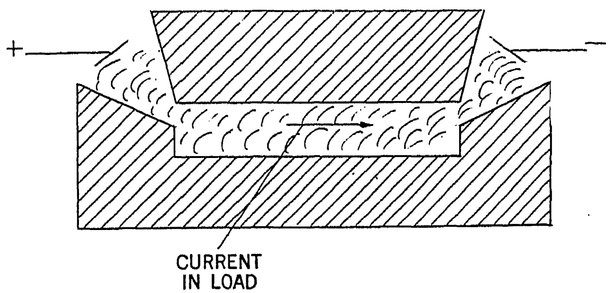


FIG. 84.—Furnace with current feeder made from same material as charge.

maintain these properties for a reasonable length of time); good mechanical strength; and low thermal conductivity (high conductivity would cause too much heat to flow from the furnace toward the outside).

Only two methods of introducing the current into the furnace are known: by means of carbonaceous electrodes; and by means of the furnace product itself. The first is the generally accepted method and the second is the exception, being limited to a few cases.

For materials whose electric conductivities do not differ too widely at low and high temperatures, the following method has been suggested. Instead of making the contact between the power supply and the mass to be heated in the hot zone of the furnace, the contact is placed near the cold outside. The material which is to be heated extends through the

wall. Of course the cross section of the material must be increased from the inside towards the outside in order to concentrate heat generation on the inside. Connection to the supply line by means of electric contacts is made near the cold zone. Figure 84 gives a schematic view of such a furnace.

As mentioned above, the only material suited for electrodes is of a carbonaceous nature. Since such material burns off at a rapid rate, the electrodes must be fed continuously into the furnace. The rate of consumption will be discussed on page 134. The method of feeding in the electrodes is always connected with the control of the electrode circuit and will be discussed on page 159.

There are two ways of using carbonaceous material: preshaped (made either of carbon or of graphite); or in the form of a green paste made of crushed carbon and a tar pitch binder. The paste is continuously baked during use in the furnace by the heat of the furnace.

Thus the following summary can be made:

METHODS OF BRINGING ELECTRIC POWER INTO ELECTRODE MELTING FURNACES

- By bringing the material which is to be heated out to the cold with increased cross section
- Carbonaceous electrodes
  - Prefabricated
    - Carbon
    - Graphite
  - Self-baking

Self-baking electrodes, employed to a considerable extent for more than 20 years in Europe, have only recently received more attention in American industry. Their use is steadily increasing.

A. Types

1. Carbon Electrodes

Carbon electrodes are made of a mixture of anthracite coal and coke with pitch used as a binder. Because of the high temperatures involved, carbon electrodes burn off at a considerable rate—about 40 lb per 1000 kwhr for steel melting furnaces. This is equivalent to perhaps 22 lb per ton. Electrodes are supplied with current by means of an electrode holder, the downward movement of which is limited by the roof. After the holder has reached the roof, that part of the electrode from the holder down to the bath cannot be used directly. Since its length in proportion to the total length of the new electrode is so considerable, it is necessary to reclaim it; this is done by connecting a new length of electrode to the short end as yet unused. Figure 85 illustrates the process: Both ends of the electrode are provided with a tapped hole; a short, cylindrical, outside threaded block (nipple) of the same material as the electrode is put into the end of the unused portion and a new electrode is screwed on.

The end surfaces of the old and the new electrodes must be absolutely plane and very clean. The thread should also be cleaned before joining the two pieces. A paste containing graphite and a binder are used to eliminate air in the connection and insure better contact.

The joints are the weakest parts of the electrode. Besides the mechanical and electrical reasons given below, it must be kept in mind that the electrodes are not homogeneous over their cross section. Because of the manufacturing process (technique of pressing as well as the heating of the electrode in the baking process), outside layers have quite different properties from layers further inside. *Mechanically*, the block must hold the two pieces together. Although the strength of the joint is entirely sufficient for mere pull, danger arises as soon as any bending stress occurs. This is one of the reasons why a straight and smooth movement of the electrode is so essential. *Electrically*, there is the problem of reducing contact resistance to a minimum. Attempts have been made to make the connecting block of metal, using the high conductivity of the metal to offset the contact resistance. This method did not work out for various reasons: different coefficients of expansion of the metal and the electrode cause cracks; in case of poor contact the metal pin gets so hot that it melts and allows the lower end of the electrode to drop into the bath. Even with initially perfect contact, as the electrode is lowered the pin heats up and finally melts, with the electrode dropping into the bath.

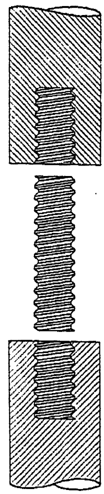


FIG. 85.—Electrode connection by threaded nipple.

There is also an additional contact resistance at the thread. Increased resistance tends to increase heat generation and consequently, by oxidation, to burn away the electrode in the region of the joint. Oxidation results in a decrease of conducting area and thus greatly shortens the life of the electrode. Actually, neither local heat generation nor oxidation occurs. That this does not happen and that the method of connecting ends works can be explained as follows: the electric resistivity of carbon decreases with increase of temperature; a local increase of temperature therefore results in a drop in resistance, which counteracts the rise of temperature. In addition, the electrode material is a good conductor of heat; locally produced heat is therefore carried away by thermal conduction. However, in order to keep this balance, great care must be used in making the joints.

There are two zones of contact resistance: one between the remaining end of the old electrode and the cylindrical block; the other between the block and the new electrode. Some of the European designers have

eliminated one of these zones by providing electrodes which have a male thread on one end and a female on the other. These threads can be made conical conveniently. Figure 86 shows such an arrangement, a design not used in the United States because the center portion of the electrodes is weaker than the outside, and the part exposed to greatest mechanical stress is thus of the weakest material.

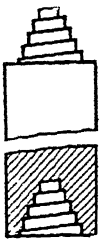


FIG. 86.—Conical male and female thread.[42a]

The physical properties of carbon electrodes are tabulated on page 125.

2. Graphite Electrodes

These are constructed in a similar but more careful way than are carbon electrodes. After baking and cooling, electrodes which are to be graphitized are subjected to an additional process—annealing at approximately 4000 F in an electric furnace. This graphitizing process takes from 7–10 days. Graphitizing furnaces are briefly discussed in Volume II.

Graphite electrodes burn at a considerably slower rate (15 lb per 1000 kwhr in steel furnaces can be considered as a typical figure) than carbon electrodes, partly because graphite electrodes can be more heavily loaded electrically than carbon electrodes and partly because of their higher oxidation temperature. Graphite electrodes are therefore smaller in diameter than are carbon electrodes for the same load. Expressed in feet of electrode length per 1000 kwhr, the consumption of graphite electrodes is about 20–25% higher than that of carbon electrodes. Hence, for graphite electrodes, using up old electrode ends is also essential. The same method of threading the ends as described for carbon electrodes is employed. Because of the smoothness and softness of graphite, however, it is not necessary to apply any paste. The smallest diameter to which threading can still be applied without danger of excessive breakage is 1.25 in.

3. Self-Baking Söderberg Electrodes

In the early 20's, the Söderberg electrode was developed in Norway, and its use has since spread considerably. Söderberg electrodes [50] are generally employed in ferro-alloy furnaces, in electrochemical furnaces (phosphate, carbide), in electrolytic processes (aluminum), in matt smelting of nonferrous materials, in electric pig-iron furnaces (as used in Scandinavia and in Finland), and to a smaller degree in steel melting furnaces. Their main field of usefulness is in medium and large sized furnaces which require electrodes from 16 or 20 in up to a maximum diameter of 8 ft.

[50] M. O. Sem, *Light Metals*, 1, 382 (1938). D. D. Howat, *Mine and Quarry Eng.*, 7, 283 (1942). M. O. Sem, *Can. Chem. Met.*, 13, 175 (1929).

In aluminum cells, rectangular Söderberg electrodes up to 4 by 15 ft are being used.

The electrode consists of a steel casing filled with a pasty mass similar to that used in the production of prefabricated and preheated carbon electrodes. The composition of the paste is different for different products and has to be selected accordingly. Under the influence of the heat produced by passage of the current through the electrode and of the heat conducted by the electrode from the furnace, the mass bakes and forms a rigid electrode. Because of the weight of the paste resting on the bottom part of the electrode, pressure is exerted on the mass which successfully replaces the external pressure needed in prefabricated electrodes. The pressure in the case of the Söderberg electrode varies with the composition of the paste, the length of the electrode, etc. A figure of 2.9 lb per sq in is typical. As the electrode is consumed, new steel sleeves are welded on top of the previous ones and new material is added inside the shell.

In order to obtain better adhesion between shell and core and to distribute the current and heat inside the mass, the steel shell is equipped with fins and ribs, as shown in Figure 87. These fins also result in a better current distribution over the entire cross section. The mass should be heated to about 250 F while being filled in. Special precautions must be taken because of considerable gas development. The gases normally escape through the shell without cracking the outside layer of the electrodes; for welded shells, holes are provided in the steel shell; and in case of riveted shells, the openings between the rivets are sufficient to permit escape of the gases. The individual steel shell, approximately 3⁄32 in thick, has a length in accordance with the diameter of the electrode.

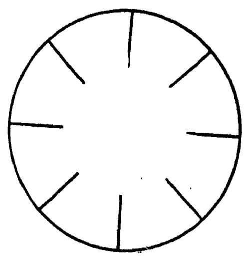


FIG. 87.—Shell of Söderberg electrode.

The manufacture of the electrode in the furnace as described above eliminates pressing of the material. The material within the steel shell will not be uniform over the entire length: the portion from the arc up to the clamp will be hard baked; the next higher zone on the electrode is half baked; while the zone near the cold end is in form of a paste or in briquettes, unchanged as fed in.

The electrodes are usually suspended near the cold end. The current is supplied by clamps located close to the furnace top, where the electrode is already almost baked. Thus the electrode can withstand the pressure necessary to insure good contact. The large and subdivided area of contact makes it unnecessary to exert extreme pressure per unit area. In order to have the electrode holder as close as possible to the

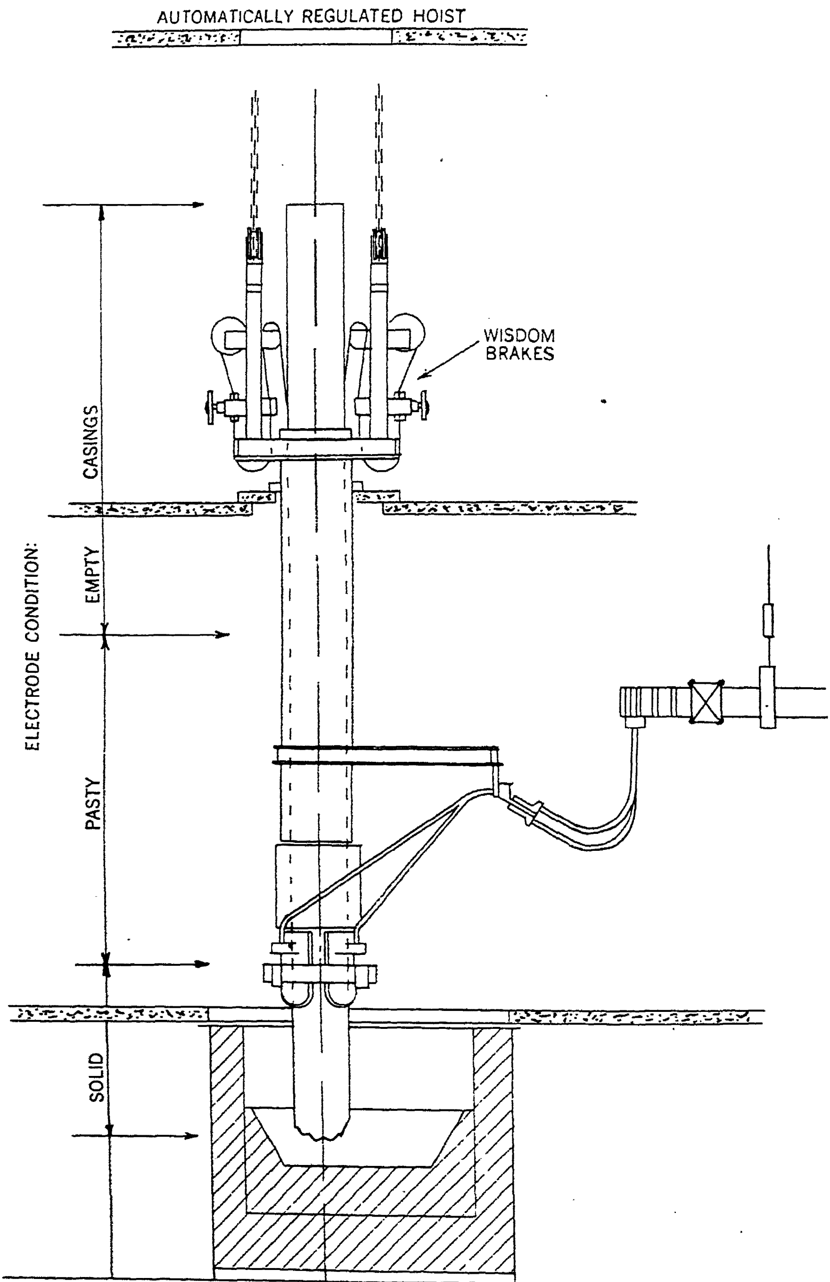


Fig. 88.—Lifting arrangement of Söderberg electrode. (Courtesy *Det Norske Aktieselskab for Elektrokemisk Industri, Inc.*)

furnace top, it must be moved at frequent intervals, although only for short distances: every time it reaches the top of the furnace, the electrode is dropped for a short distance. In this connection the introduction of "Wisdom brakes" was a definite step forward, making possible easy releasing of the electrodes in small steps. Figure 88 shows the present method of suspension and at the same time the way in which the electrode is prolongated. The electrode is held, lowered, and lifted by means of a suspension operated above a working platform. If the electrode is burned off so far that the water-cooled supply ring comes very close to the roof of the furnace, the tension on the strip brakes (Wisdom brakes) is released by means of hand wheels. When the tightening bolts on the clamps are loosened, the electrode slips downward under the influence of its weight, which is sometimes sufficient to move the electrode downward even without loosening of the bolts. After lowering the electrode, the brake is easily tightened again and operation continues. It is not necessary to shut off the power during this operation, which lasts only approximately 5 minutes. When the electrode is lowered far enough, a new part of the shell is welded on top of the old one.

This description applies only to arc furnaces. Aluminum furnaces have Söderberg electrodes of quite different design.

4. Composite Electrodes

For large furnaces, in which even the largest diameter electrodes are not sufficient, composite electrodes are used. In the simplest form, a number of rectangular blocks form one rectangular electrode (Fig. 89).

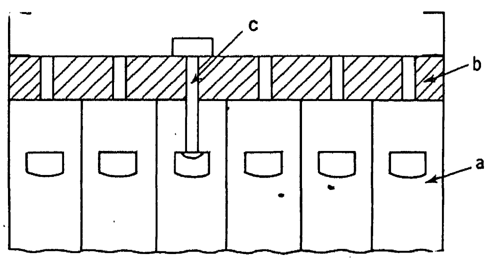


FIG. 89.—Rectangular electrode composed of six individual blocks (*a*); (*b*), base plate to which current supply is attached. The individual blocks are attached to the base by bolts (*c*).

The (French) Miguet electrode[51] was developed principally for single-phase furnaces. In order to obtain large round electrodes, carbon pieces of segment form are combined around a steel frame (Fig. 90). The blocks are bolted on the inside of the structural steel. The blocks in the

[51] M. Arrouet, *Trans. Electrochem. Soc.*, 52, 335 (1927).

various vertical rows are connected by dovetail contact. The center of the electrode is filled with pasty material, which does not essentially participate in the conduction of current but serves mainly to eliminate oxidation by air. The steel frame relieves the main part of the electrode

FIG. 90.—Arrangement of Miguet electrode.[48a]

carbon from mechanical stress. The frame is insulated from the carbon because the frame is not supposed to participate in current conduction. The lowest carbon blocks, those reaching directly into the furnace, are held by the pressure of the electrode clamp. The Miguet electrode is used only in large sizes and only in large electrochemical furnaces.

B. PROPERTIES

Table XI gives the approximate values of various properties of carbon and graphite electrodes. (Because of the composite nature of the Söderberg and Miguet electrodes, the properties as listed below for the homogeneous carbon and graphite electrodes would be meaningless for the self-baking and composite types.) In apparent density and weight, the different values hold for different diameters, the strength decreasing with greater thickness. The coefficient of thermal expansion at Fahren-

heit temperature (t) equals $K + 0.0039\,t \times 10^{-7}$, and at Centigrade temperature, $0.8\,K + 0.007\,t \times 10^{-7}$.

Thermal and electric conductivities change greatly with temperature. Carbon electrodes have a lower thermal conductivity at elevated temperatures than at low temperatures:[52] at 212 F (100 C), $k = 4.6$ Btu per ft, hr, F (C); and at 2912 F (1600 C), $k = 2.66$. Carbon refractories

TABLE XI

PROPERTIES OF CARBON AND GRAPHITE ELECTRODES

Properties	Carbon electrode	Graphite electrode
Apparent density	1.54–1.55	1.53–1.56
Weight, lb/cu ft	95–96	95–97
Tensile strength, lb/sq in	400–660	750–1900
Compressive strength, lb/sq in	1900–2900	3100–6000
Transverse strength, lb/sq in	800–1300	1500–3800
Thermal expansion, cold, K =	12–13	5–12

have conductivities of roughly one-half of these values. The thermal conductivities of carbon as well as of graphite are considerably different for flow in the longitudinal and in the transverse direction. Figure 91 shows curves for the thermal conductivity of graphite as a function of

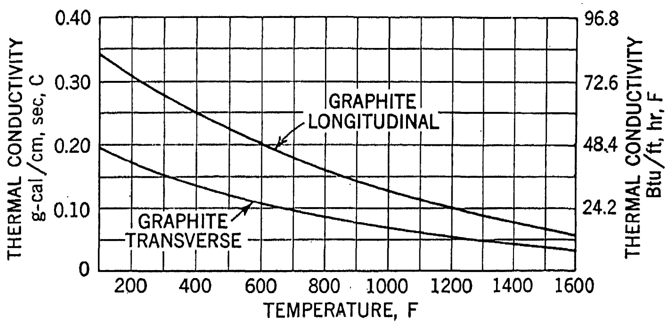


FIG. 91.—Thermal conductivity of graphite.[52a]

temperature. The thermal conductivity of the Söderberg and the Miguet electrodes cannot be readily expressed by a curve. In both cases the electrodes are complex structures in which parallel heat flow occurs. In the Söderberg electrode, the conduction near the outside surface is proportionally larger in small electrodes than in large ones; but this is somewhat offset by the ribs and reinforcements conducting heat from the casing to the center.

The *electric resistivity* of graphite electrodes (graphitized coke) decreases with increase of temperature, until it reaches a minimum value

[52] F. J. Vosburgh, *Trans. Am. Inst. Elec. Engrs.*, 52, 844 (1933).

[52a] F. J. Vosburgh, *Steel*, 112, April 5, 106 (1943); April 12, 118 (1943).

at approximately 1000 F. Beyond this value, the resistivity increases again and reaches, at approximately 3200 F, the same value as at room temperature. Figure 92 shows the change of resistivity plotted *vs.* temperature, the value at 0 C (32 F) representing 100%. The figure shows three curves: one for graphitized coke, one for coke carbon, and one for

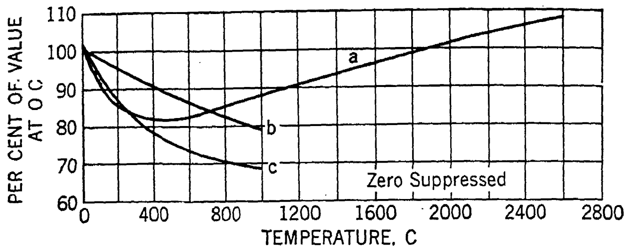


FIG. 92.—Electric resistivity of graphite and carbon: *a*, graphitized coke; *b*, coke carbon; *c*, coal carbon. (Courtesy *National Carbon Co.*, New York, N. Y.)

coal carbon. The latter two are plotted only up to 1000 C (1832 F); at higher temperatures, carbon starts to graphitize and the resistivity curve gradually approaches that of graphitized coke. At room temperature, the resistivity of electrodes is a function of the electrode size, as may be seen from the figures in Table XII.

TABLE XII

RESISTIVITY OF ELECTRODES AS A FUNCTION OF SIZE

Type of electrode	Diameter size, in	Resistivity, ohm × in
Carbon	Up to 14	0.0013
	Up to 17–24	0.0014
	24–40 (and square 24 × 40)	0.0026
Graphite	Less than 3	0.00033
	3–8	0.00036
	9–12	0.00037
	14–18	0.00039

The electric resistivity of the Söderberg electrode cannot be readily expressed by a single numerical value. The steel casing and the ribs carry a great part of the electric current; the composite nature of the electrode makes the influence of the skin effect still more complicated. No figures for the electric resistivity of the Söderberg electrode are available. The electric resistivity of the Miguet electrode is that of the individual carbon blocks, because the steel parts are not meant to participate in carrying the current.

Specific heat is of course also a function of temperature. The figures in Table XIII are for the mean specific heat (in Btu/lb, F or cal/*g*, C) between the temperatures shown.

TABLE XIII

SPECIFIC HEAT OF ELECTRODES

Degrees C *Degrees F*	26–76 79–169	26–282 79–540	26–538 79–1000	39–902 102–1655	47–1193 117–2179	56–1450 133–2642
Carbon	0.168	0.200	0.199	0.315	0.352	0.387
Graphite	0.165	0.195	0.234	0.324	0.350	0.390

C. ELECTRODE CONTACTS

Electrodes must be suspended in all types of furnaces; furthermore they must obviously receive current. Usually the electrode holder and the current-supplying clamp are combined in one unit. Very large electrodes, especially in ferro-alloy and electrochemical furnaces, have a

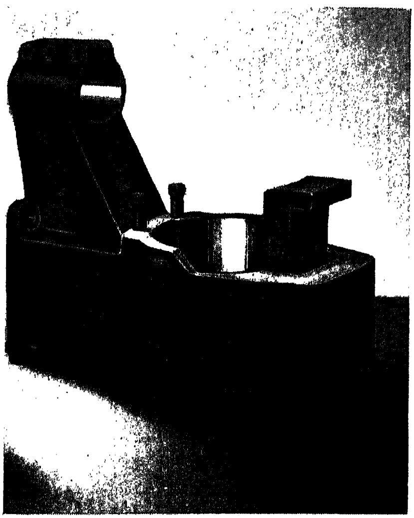


FIG. 93.—Electrode holder for steel furnaces. (Courtesy *American Bridge Co.*, Pittsburgh, Pa.)

holder separate from the current-supplying clamp. The holder also serves to lift and lower the electrodes for purposes of control and for moving the roof. Except for very small types of arc furnaces, almost of laboratory size, all electrode holders are water cooled. This is considered essential for having a good contact and preserving the holder.

The electrode holder itself is almost always made of copper, a very smooth inner surface insuring good contact. A wedge holds the electrode in position, and the body of the holder is hollow for water cooling. Figure 93 shows such a *one-piece holder* with a heavy wedge. In Europe, holders are sometimes made of steel, the contact resistance of steel, while slightly higher than that of copper, changing less when the holder oxidizes. Furthermore, steel holders of moderate size need not be water cooled. This is a typical and interesting example of the difference between European and American furnace design. The American design calls for less supervision but causes greater energy loss (through water cooling) than does the European design.

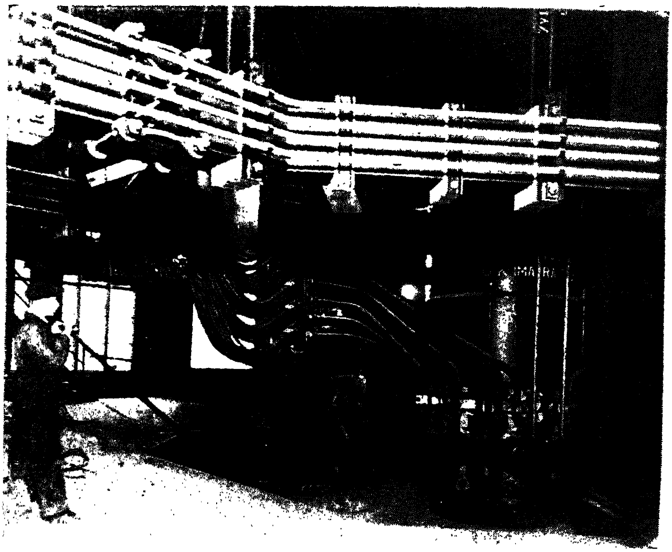


FIG. 94.—Electrode holder for large electrochemical furnaces. (Courtesy *Det Norske Aktieselskab for Elektrokemisk Industri, Inc.*)

Larger electrodes, especially in electrochemical work, are held in *ring holders composed of several pieces.* The individual pieces are pressed against the electrode either by bolts (Fig. 94) or by steel springs released by compressed air.

The electrode and the holder must be adjustable as a unit. The main direction of movement is of course the upward and downward movement used in controlling the input by changing the length of the arc. Small adjustments are necessary in order to safeguard the fitting of an electrode into the hole of the electrode ring. This calls for adjustment in the two directions of a horizontal plane. It is good practice to make these adjustments, not at the electrode holder, but back where the holder

is connected to the arms. The holder as shown in Figure 93 is connected by a tubular arm to a column at the side of the furnace.

In some instances the weight of the electrode is counterbalanced. Figure 95 illustrates the general arrangement of one electrode, electrode holder, electrode arm, crosshead, and column. The power necessary for

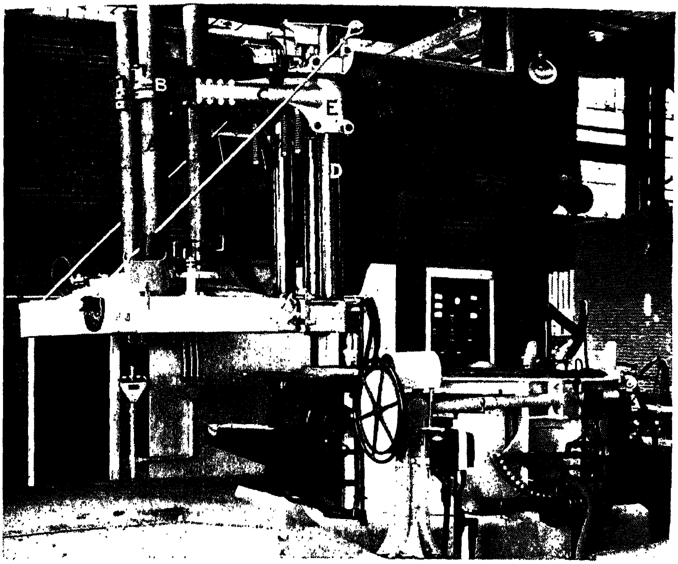


FIG. 95.—Assembly of electrode, electrode holder, arm, etc. *A*, electrode; *B*, electrode holder; *C*, tubular arms; *D*, columns; *E*, crosshead. (Courtesy *Pittsburgh Lectromelt Furnace Corp.*, Pittsburgh, Pa.)

the electrode travel is not great. But in order to obtain sensitive control it is imperative that the movement be carried out without appreciable friction and very rapidly. This accounts for the relatively complicated arrangement. The movement on the column is controlled by motors, usually d-c (for electrode control, see page 159).

D. Length of Electrodes

The electrode length is selected within relatively wide limits. As referred to here, "length" means the distance from the hot end to the end in the electrode holder. It usually includes at least one and sometimes more joints. The lower limit of length is given by the distance from the bath to the top of the roof plus the necessary length for the clamping device. If the electrode were no longer than that, it could never be lowered by the automatic control. The upper limits are given by the necessity of careful handling and space above the furnace.

The longer the electrode, the less frequent are the electrode changes and the accompanying inevitable delay in production. The entire electrode energy balance, which also influences the maximum power obtainable from the furnace, is a function of the length. This balance and the operation of the control change as the electrode burns off. For very long electrodes, the characteristics of control and the heat balance at the beginning of the electrode life and at its end may be entirely different. Long electrodes are heavy and awkward to handle. Moreover the energy losses caused by the electrode increase with its length. The heavy weight of long electrodes makes the control less efficient. Lengths are not used which are greater than the maximum fabricated length plus the distance from the electrode holder (if sitting directly on the electrode ring) to the bath (Fig. 96). The factors influencing selection of electrode length are given in Table XIV.

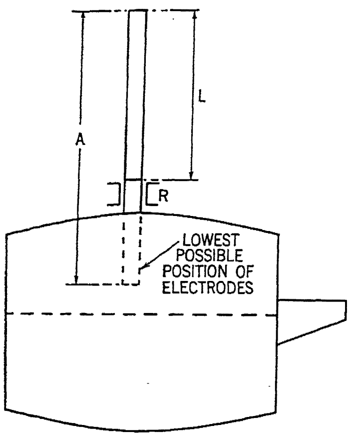


FIG. 96.—Length of electrode. *L*, fabricated length of electrode; *R*, roof ring. The maximum length, *A*, equals the manufactured length of the electrode plus the distance from the top of the roof ring to the lower tip of the electrode.

The lengths of Söderberg electrodes can be held rather small: thus the advantages of short electrodes may be enjoyed without the disadvantages mentioned above (because the electrode is continuous). However, a relatively large length of electrode will extend beyond the holder. This extension does not carry current but increases to some extent the energy losses: it acts as a cooling fin for the conducting part and in addition increases the weight and consequently lowers the sensitivity of the control.

TABLE XIV

FACTORS INFLUENCING SELECTION OF ELECTRODE LENGTH

Points favoring small length	Points favoring great length
More rapid response of the control	Labor saving because of less frequent adding of new electrodes
Easier handling because of small weight	Less frequent interruption of furnace service
Small changes in operational behavior between lowest and highest position of the electrode	
Smaller energy consumption	

E. Selection of Area

1. Graphite and Carbon Electrodes

The cross section of electrodes is selected in accordance with recommendations of electrode suppliers. For graphite and carbon, a certain current density is recommended as maximum for each electrode diameter. Table XV records the values for carbon electrodes as recommended by

Table XV

Current Densities for Carbon Electrodes

Diameter of round electrode, in	Side of square electrode, in	Current density, amp/sq in
Up to 12	Up to 12	40–60
14–24	14–20	35–55
	24	35–45
30		35–50
35–40	24–30	30–40

the *National Carbon Co., Inc.* For graphite electrodes, Figure 97 shows the maximum and minimum values of current density plotted *vs.* diameter and side. Each recommended density holds for rather wide limits;

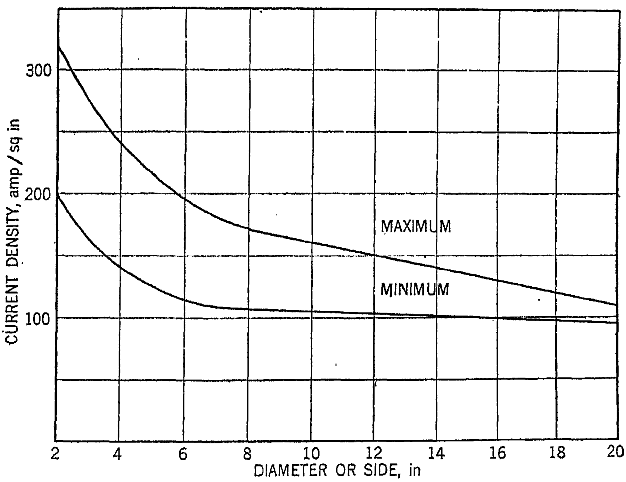


Fig. 97.—Maximum and minimum values of current densities for graphite electrodes. (Courtesy *National Carbon Co., Inc.*, New York, N. Y.)

this practice is justified because the selection of the diameter (or side) of the electrode is still entirely empirical.

The selection of area has been described in general on page 57. The diameter or side of the electrode should be so selected as to give

minimum energy losses. These losses depend upon the following items: current passing through the electrode; frequency of the electric current; electric conductivity of the electrode at various temperatures; shape and dimensions of the electrode, including length; distance between the arc and roof; thickness of the roof; temperature at the foot of the arc; temperature of the furnace chamber; temperature of the electrode ring in the roof; temperature of the air surrounding the top of the roof; and thermal conductivity of the electrode at various temperatures. The *frequency* of the current influences the skin effect and therefore the resistance (a-c) of the electrode; if the electrode is considered to be a cylinder, the influence of the frequency on the skin effect can be read from Figure 104 (page 144); at 60-cycles frequency, the influence of the skin effect is small. *Electric conductivity* is a function of temperature and of course of material (carbon or graphite); see page 126. The electrode *shape* is cylindrical or rectangular; under the influence of the elevated temperatures of the arc and the air in the furnace, the bottom burns off, and after a short time the part in the furnace (*i. e.*, from the roof ring downward) becomes conical. The *length* also changes. The *distance between arc and roof* changes as the control raises and lowers the electrode in order to obtain various lengths of the arc; the distance depending also on the condition of the charge and on the level of the charge (high or low). The *temperature at the foot of the arc* is the same for graphite and for carbon electrodes, and probably does not change with the load or any other condition of the furnace. The *temperature facing the electrodes* (furnace chamber, electrode ring, air) influences the heat exchange between electrodes and surroundings. *Thermal conductivity* is a function of temperature and of course material (carbon or graphite); see page 125.

The conditions in the electrode are described in the following three sections.

(*a*) *Portion of Electrode from Arc to Roof*

Because of the skin effect, the current will be carried mainly in the outside layer of the electrode. The heat due to the ohmic losses will therefore be generated in the outside layer; from there, part of the heat will flow radially toward the core. The entire cross-sectional area will participate in the conduction of heat from the bath towards the water-cooled ring in the roof. Near the arc, the core of the electrode will be considerably hotter than the surface. With increasing distance from the arc, temperature uniformity over the cross section will improve. If the electrode is very long, the temperature distribution in the upper end (*i. e.*, near the roof) may be reversed, the temperature in the core being lower than the temperature on the surface. The trend of the resistivity–temperature curve makes the skin effect in this part less marked.

(*b*) *Portion of Electrode in the Roof*

The electrode ring surrounding the electrode is water cooled. At this point, the temperature of the surface of the electrode is materially lowered.

The influence of cooling is marked in the immediately adjacent parts of the electrode, the core of the electrode probably being hotter here than at the corresponding outside areas.

(c) *Portion of Electrode above Roof, below Holder, Exposed to Air*

In this part of the electrode, there probably does not remain much of the heat conducted from the bath. On its outer side, however, heat is being generated by the current. The temperature coefficient of resistivity causes a still more unevenly distributed electric load: the skin effect tends to push the current towards the outside; and the negative temperature coefficient results in lower temperature on the inside and consequently higher resistivity. Thus the current is even more crowded toward the outside.

As the electrode burns off, it changes shape, the lower part becoming conical. The smaller diameter causes a less marked skin effect and therefore retards a further decrease in diameter.

In view of the conditions described it may be well worth while to consider hollow electrodes. These would weigh much less than solid electrodes. Although their replacement cost probably would not be less than that of solid electrodes, their energy consumption would be lower.

2. Söderberg Electrodes

The current density of the Söderberg electrode can, because of the steel shell, be higher than that of the equivalent section of a carbon or a graphite electrode. The current density should again be given as a function of electrode size, because the perimeter, which determines the resistance of the steel shell, is proportional to the diameter, while the cross section is proportional to the square of the diameter. Unfortunately, only one figure seems to be available: this shows a specific current density of 90 amp per sq in without specifying the diameter.

The flow pattern in this type of electrode is more complicated than in carbon or graphite electrodes. The heat generated in or conducted into the material is used in part to sinter and bake the material. This part of the heat can therefore not be considered as energy loss. The heat flow in itself is complex because of the parallel streams in the shell, the ribs on one side, and the filling on the other.

F. Electrode Consumption

Since electrode consumption is influenced by a large number of individual factors, figures differ within considerable limits. The maximum and minimum ratio of reported figures for the same product is frequently 1:2, but ratios as high as 1:6 occur.

Factors influencing the amount of electrode consumption are: the temperature of the bath; time of holding at temperature; furnace atmosphere conditioned by the degree of airtightness; size of furnace; mode of

operation—continuous or otherwise, number of heats per day; current density; nature of charge and size of individual pieces (possibly causing breakage when electrodes settle on charge); kind of finished product; and, last but not least, the human element.

Table XVI [53] contains data showing the lower limit, the upper limit, and the average electrode consumption for various applications. In the last column, the consumption of carbon and graphite electrodes is compared, the ratio of the average value for both types being given. The wide range of electrode consumption for each individual application indicates that it would be well worth while to make systematic studies to determine quantitatively the effect on electrode consumption of different designs, modes of operation, etc.

TABLE XVI

CONSUMPTION OF CARBON AND GRAPHITE ELECTRODES [a]
(Expressed in lb/1000 lb product)

Application	Carbon	Average	Graphite	Average	Carbon-graphite ratio
Nonferrous metals	—	—	1.25–5	3.12	—
Iron castings, cold scrap	—	—	2.50–5	3.75	—
Same (duplex, hot charge)	—	—	1.50–7	4.25	—
Steel castings	4.5– 10	7.25	2.25–5	3.62	2
Steel ingots	8.5– 15	11.0	5–10	7.5	1.57
Ferrosilicon, 15%	10 – 22	16.5	—	—	—
50%	22 – 33	27.5	—	—	—
80%	35 – 52	43.5	—	—	—
80% Ferromanganese	—	31.2	—	—	—
High-silicon ferrosilicon	45 –145	90	—	—	—
Ferrochromium	—	45	—	—	—
Calcium carbide	15 –100	57.5	—	—	—
Phosphorus	—	30	—	—	—
Iron ore (production of pig iron in the electric furnace)	9 – 13.5	11.25	—	—	—

[a] F. J. Vosburgh, *Trans. Am. Inst. Elec. Engrs.*, 52, 844 (1933).

G. CARBON *vs.* GRAPHITE ELECTRODES

For certain applications, metallurgical or chemical reasons dictate the use of graphite electrodes exclusively, while in others, for reasons of the same nature, only carbon electrodes can be used. In certain fields there is a tradition favoring exclusively the one or the other kind. For instance, ferro-alloy furnaces are almost always run with carbon electrodes and nonferrous furnaces are operated exclusively with graphite. The melting of iron and steel is one extensive application in which both kinds of electrodes may be encountered (although most new furnaces are

[53] F. J. Vosburgh, *Trans. Am. Inst. Elec. Engrs.*, 52, 844 (1933).

equipped with graphite electrodes). But here, as well as in those cases in which tradition governs the selection of the type, it is well to investigate the advantages of both. Of course it is a general practice so to select the electrode as to obtain smallest over-all cost. The electrode has a direct bearing on the following cost items: cost of electrode replacement; energy consumption; labor; first cost and depreciation; and rejections.

Cost of Electrode Replacement.—This is, of course, the product of electrode consumption (in lb), output, and price (per lb). In the United States, the price per pound of graphite is approximately twice that of carbon. (In Europe, the price ratio is approximately 6:1. This different price ratio accounts for some of the differences in arc furnace practice between Europe and America.) The ratio of 2:1 holds f.o.b. the manufacturer's plant. For hauling long distances, the price ratio may be greatly changed because of the smaller freight charges for the smaller, thinner, and lighter graphite electrodes. If the cross-section of the electrode is so selected that the graphite consumption is half that of the carbon consumption, the cost of electrode replacement would be equal for graphite and carbon. If the ratio of consumption changes in one or the other direction, the cost of replacement changes accordingly. No figures have been published showing electrode consumption (either with graphite or with carbon) as a function of load.

Energy Consumption.—As discussed above, the selection of area and of current density should be based on the total energy balance, including heat and electrical losses. For each electrode load, (amp per electrode) there is one diameter which yields the smallest energy losses. It is highly improbable that this minimum is the same for graphite as for carbon electrodes. For example, analysis might show that, in a certain case, the minimum losses with graphite electrodes are 25 kw with 12-in electrodes; the minimum losses with carbon electrodes in the same case may be 21 kw with 17-in electrodes. So far, no systematic analysis of energy consumption by electrodes has been made.

Labor Cost.—This factor must be considered because of the weight of the electrode. Furnaces in which electrode consumption is high are suitable for graphite electrodes. For a current of 20,000 amp, a 24-in carbon electrode would, for example, have to compete with a 14-in graphite electrode. A 24-in carbon electrode, 72 in long, weighs 3200 lb. A 14-in graphite electrode, 60 in long, weighs approximately 500 lb; if 72 in long, it weighs 600 lb. When frequent handling is necessary, the smaller weight of graphite may result in appreciable labor savings.

First Cost and Depreciation.—The greater weight of carbon electrodes (having larger diameter) calls for heavier electrode arms and heavier structural design, which of course raises the furnace price and

therefore the depreciation. This influence is marked for heavy currents. For low currents, the savings through small structural weight are unimportant.

Rejections.—Two items enter the picture: Lighter electrodes make control of the electrode better and more responsive, thus favoring graphite electrodes; and chemical or metallurgical demands may restrict the use of one or the other of the two materials to the point of practically eliminating one of them. For example,[54] in steel practice carbon electrodes make slag more easily than graphite electrodes. In fact, some operators claim that carbon electrodes tend to make their own slags. This can probably be attributed to the dropping of anthracite particles from the electrodes into the slag. Graphite electrodes tend to bore through the charge because of their smaller diameter. In ferro-alloy furnaces, a large effective working surface is essential to efficient production, and is best obtained with carbon. It is probable that, in ferro-alloy operations, a considerable proportion of the consumption is lost by abrasion by the charge as it passes down along the surface of the electrodes. Such a loss would be little effected by the type of electrode.

Refractory Life.—Graphite electrodes tend to dig holes into the furnace bottom. Carbon electrodes, because of their larger diameter, have an umbrella effect over the arc and protect the roof better than the smaller graphite; but graphite electrodes, being smaller in diameter, keep the arc further from the sidewalls and perhaps to some extent save them.

Mechanical Strength.—The diameter which yields smallest energy consumption may be undesirable from the viewpoint of mechanical strength. In small and in medium sized furnaces, the diameter of graphite electrodes which gives the smallest energy consumption may be so small that danger of excessive breakage may occur. For mechanical reasons, a heavier electrode may then be selected, which would upset the comparative figures for carbon and graphite. Breakage of graphite electrodes is always higher because of the smaller diameter used. On the other hand, graphite joints are much easier to make, not only because no joint compound is required, but because the electrodes are considerably lighter and easier to handle. Graphite electrodes are smooth and harder to hold in the holders, but they are turned to be a true cylinder and so are less troublesome to pass through the holders than the rough, less cylindrical, carbon electrodes.

IV. BUSSES (CONNECTORS)

The largest connected load so far used in one group of three electrodes in arc furnaces for the melting of steel is 18,000–19,000 kva. The highest potential used in such furnaces is 325 v. The maximum current

[54] F. J. Vosburgh, *personal communication.*

in one phase is therefore approximately 40,000 amp. Obviously, so large a current calls for a very careful design of the current-carrying elements. In fact, as will be shown in the section on diagrams (page 170), the maximum power in a furnace is limited by factors inherent in a successful design of the busses.

The current-carrying system consists mainly of three parts: busses (rigid) bringing the current from the transformer terminals to the vicinity of the furnace; cables (flexible) connecting the busses to the electrode columns; a current-carrying system over the top of the furnace, connecting the flexible cable with the electrode holder. Flexible cables are necessary because the electrodes are moved up and down by the control device because of tilting, and in top-charge furnaces because of movement of the roof. In large ferro-alloy furnaces, which are not tiltable and have no roof or a fixed roof, the flexible cables can be made shorter and simpler.

The current-carrying system causes energy losses and a change in phase angle (inductive voltage drop). The ohmic and the inductive voltage drop are of considerable importance in connection with the stable operations of the furnaces and with the maximum power input. It is therefore important to calculate as closely as possible the resistance and the reactance. Theoretically, a multiple problem and choice are involved: the selection of a desired voltage drop and energy loss and then the selection of cross section and the arrangement of the busses. In practice, however, this choice is rarely considered, and the tendency is to load the busses as heavily as possible, *i. e.*, as heavily as their temperature increase within safe limits will permit. In large furnaces, the electrodes are arranged to give the smallest inductance. In small furnaces, a relatively high inductive resistance in the busses is desirable.

A. Rigid Busses

Copper is almost the only material used for furnace busses. (Aluminum is used occasionally, especially abroad.) Slight impurities in the copper, unavoidable in industrial manufacture, increase the resistivity above the value for pure copper. The electric resistivity of bus copper is 0.6789×10^{-6} ohm $\times$ inch at 20 C. The resistivity increases with temperature. The temperature coefficient of bus copper (α) is 0.00393 per degree C. The actual resistivity, R_a, in ohm $\times$ inch, at a temperature of t degree C is:

$$R_a = 0.6789 \times 10^{-6} [1 - \alpha(t - 20)]$$

Since the heavy currents necessary in arc furnaces can rarely be carried in one bus, several parallel conductors are used. To increase the current-carrying capacity and to decrease the proximity effect (see page

63) and mutual induction, either an interweaving of the busses or other special arrangements is used. Figure 98 shows some possible arrangements, in which all arrangements refer to one phase and are taken three times for three phases, the three phases being either in one plane or in triangular grouping. Of course any deviation from the direct and simple design complicates the mechanical arrangement and the insulation.

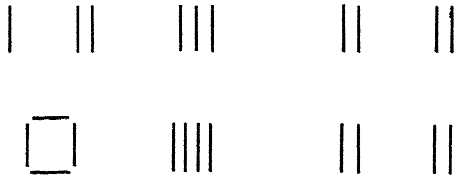


FIG. 98.—Bus arrangements.

In this section, the permissible load for various groupings of busses will be discussed. Ohmic and inductive resistance will be considered on page 142.

For low currents, up to 1000 or 1500 amp, a general rule—"1000 amp per sq in cross section of the bus"—can be applied. For higher currents, this formula is not valid: It is necessary to split the busses, *i. e.*, to subdivide the total cross-sectional area. In so doing the surface (cooling area) is increased, the skin effect is diminished, and consequently a higher load per unit volume of copper can be applied. The skin effect was mentioned on page 63. In addition to the skin effect, the proximity effect is involved. Pabst[55] gives some experimental data on permissible loads for currents up to approximately 6000 amp, but only for single phase systems. Since the proximity effect is better in three-phase systems, the loads indicated by Pabst are on the safe side for three-phase current.

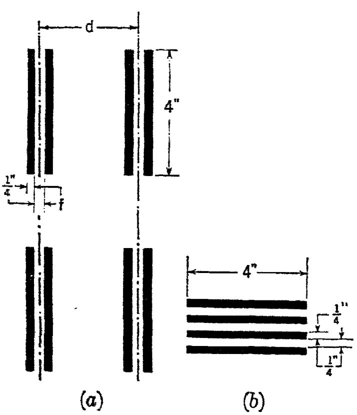


FIG. 99.—Comparison between two bus arrangements with same total amount of copper. In Fig. *a*, two sets (for two phases) of busses are shown; only four of the total of eight straps belong to one phase (either left or right half of arrangement).

A black paint applied to the surface increases the dissipation of heat and, in certain cases, raises the permissible load by 10%. It is important to arrange the busses so that air circulation is not hindered. The arrangement shown in Figure 99*a* has a permissible current of only 2200 amp

[55] H. W. Pabst, *Elec. World*, 94, 569 (1929).

(35 C temperature rise) as against 4200 amp for the arrangement of Figure 99*b*. Although in both instances the same amount of copper is used, in the arrangement of Figure 99*a* the flow of air is virtually impossible, while the arrangement in Figure 99*b* results in a "stack effect." If applied to a three-phase system, the arrangement in Figure 99*b* results also in much better utilization of the copper because of the proximity.

It is desirable to choose the spacings between the various busses so that the load per bus is as uniform as possible. In order to accomplish

FIG. 100.—Arc furnace on platform. Note the clamps for keeping the cables together. (Courtesy *American Bridge Co.*, Pittsburgh, Pa.)

this, bars of the same phase should be spaced as closely as possible (*f* in Fig. 99*a*) and the spacing between the two phases should be as large as possible (*d* in Fig. 99*a*). For busses of 6000 amp or more carrying capacity per phase, the spacing between the phases should be 18 in or more.[56]

For large furnaces, hollow, water-cooled tubes are used as busses, an arrangement having a twofold advantage. Because of the skin effect, only the outer part of a conductor is effective in carrying current. The use of a hollow core for water cooling does not impair the current-carrying

[56] C. C. Levy, *Trans. Am. Inst. Elec. Engrs.*, 51, 903 (1932).

capacity. In addition, the water carries heat away. Water-cooled hollow tubes are especially used for the connections between the flexible cables and the electrode holders. They must of course be carefully insulated from the steel structure of the roof and of the furnace shell.

B. Flexible Connectors

The cables used as flexible connectors are rope lay strands composed of several members, each of which may be made of some 700 to 1200 individual wires, 60 to 40 mils thick. The individual cable consists of perhaps 30 to 60 such members, resulting in a total cross section of 2,000,000 circular mils. Cables of greater size would be very stiff and are therefore generally not used. Several cables in parallel must be employed for currents higher than those which can be carried by one such cable. This again causes a proximity effect and inductive voltage drop. Such groups of cables are held together by large insulated clamps (as may be seen in Figure 100), making the cables fairly stiff. The cables of the various phases must be spaced sufficiently far apart to eliminate any danger of shorting. In some instances, water-cooled cables have been

Table XVII

Size of Copper Cable for Various Transformer Sizes

Transformer capacity, kva	Million circular mils per phase	
	Water-cooled cable	Air-cooled cable
1000	Not used	4.2
2000	Not used	10.5
4000	6.0	22.5
6000	8.0	33.2
8000	10.1	43.0
10000	12.0	50.5
12000	13.2	56.0

used: A number of relatively small-diameter cables are arranged around a core and cooled from the outside by a rubber hose. The necessary cross section of copper has been decreased to 25% of the cross section needed without water cooling,[57] as may be seen from Table XVII.

C. Contact Resistance

In view of the heavy load on busses, the problem of contact resistance is of prime importance. A misconception frequently encountered is that the specific pressure (lb per sq in of contact area) has a direct bearing on the resulting contact resistance. Numerous investigations have

[57] T. J. Ess, *Iron Steel Engr.*, 21, AF7 (1944).

proved that the contact resistance is determined by the total force and of course by the nature and surface condition of the conductor metals. Merely increasing the surface therefore does not decrease the contact resistance. It is especially useless to increase the length of the contact.[58] If the surface is to be changed at all, the width should be increased. Figure 101 illustrates this. Donati shows that, at one-half

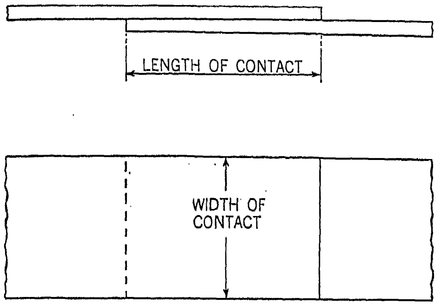


FIG. 101.—Contact zone of bus bars.

the length of the contact, the maximum current density occurs, the amount of which is independent of the length of the contact. Hoepp[59] investigated systematically the contact resistance of busses. Figure 102, taken from his investigations, plots contact resistance *vs.* total contact pressure in pounds. Curve *b* shows higher contact resistance due to lack of tinning even though the contact area is larger. For a theoretical proof of this behavior, see Holm and Holm.[60]

Contact resistance changes rapidly with temperature. Copper forms a thin layer of oxide at relatively low temperatures. Since copper oxide has a very low conductivity, contact surfaces are therefore frequently tinned and sometimes silvered to avoid local oxidation. (An extreme increase of temperature would theoretically decrease the contact resistance: the metal would become soft enough to cover all rough parts and eventually would melt. Such contacts are of course technically unusable.) Greasing the contact surfaces with pure vaseline is also good practice. At the points of real contact, the vaseline is squeezed away, or no contact could be obtained because vaseline is a fairly good insulator. However, it will still remain on those parts of the surface which, because of microscopic unevenness, will not give metallic contact. If not covered by vaseline (or oil), these parts are exposed to air and start to oxidize, gradually contaminating neighboring sections.

[58] E. Donati, *Energia Elettrica,* 12, 433 (1935).

[59] W. Hoepp, *Elektrotech. Z.*, 41, 205, 232 (1920).

[60] E. Holm and M. Holm, *Z. tech. Physik,* 3, 290, 320, 349 (1922); 6, 166 (1925); 7, 198 (1926); 8, 141 (1927).

D. Determination of Resistance and Reactance

The entire behavior of an arc (and an arc resistance) furnace is determined by the resistance and reactance of the electrical system. It will be shown (pages 170–195) that the resistance and reactance of the bus system are decisive for the efficiency and the maximum power of a furnace. In view of their importance, it is surprising how little information about them is available.

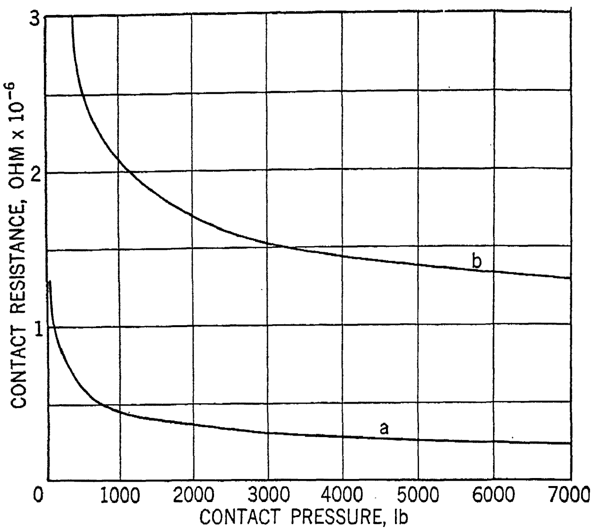


Fig. 102.—Contact resistance *vs.* contact pressure: *a*, contact with an area of 2.2 sq in (16 sq cm), the contact of area being tinned and oiled; *b*, contact of 4 sq in (26 sq cm), the surface of contact being cleaned and oiled (after Hoepp).

It would be desirable to calculate the resistance and reactance of the entire system as part of the engineering calculation of the entire furnace. Unfortunately, because of the complexity of the problem, exact formulas are available only for certain simplified conditions. Measured data from actual installations have not been made public. As long as no formulas are available which will permit with reasonable accuracy and simplicity the calculation of resistances and reactances, it is highly desirable to analyze systematically actual furnace installations, not only obtaining the over-all reactance and resistance values, but breaking these down for the various parts of the current path. In the following, the available formulas and methods of calculation are briefly reviewed.

1. Resistance

(a) *Rectangular Busses*

Walter[61] and Dannatt and Redfearn[62] have developed a general formula for a-c resistance of individual rectangular busses. The ratio σ_α of a-c resistance to d-c resistance depends upon a characteristic figure called the attenuation factor. Symbolically, this is written $\beta \cdot d$, where $\beta = 2\pi\frac{\sqrt{\mu f}}{\rho}$, d is the thickness of the bus material, μ is the permeability of the bus material, f the frequency, and ρ is the d-c resistivity. In Figure 103, $\sigma_\alpha = R_{a\text{-}c}/R_{d\text{-}c}$ is plotted *vs.* $\beta \cdot d$. For copper, $\beta = 1.069\ \text{cm}^{-1}$.

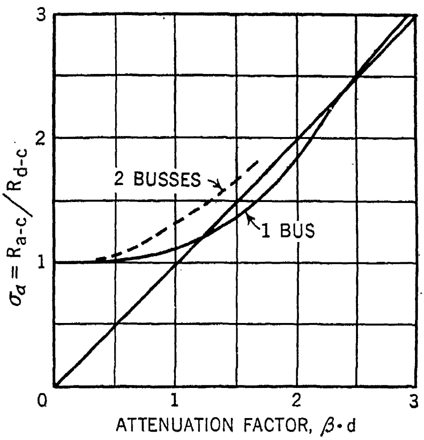


FIG. 103.—Resistance increase *vs.* attenuation factor for rectangular busses.

Consequently, a thickness of 1 cm (0.4 in) can be safely applied. The curves hold for layouts in which the return bus is very close to the outgoing bus.

If the return lead is practically at "infinite distance" from the outgoing bus, the copper thickness should be doubled. For instance, $\sigma_\alpha = 2$ will be reached only at a value of $\beta \cdot d = 4.25$. For various finite distances, the value of σ_α will be between the two limits given above (*i. e.*, between the value for "return bus very close to outgoing bus" and "return bus at infinite distance from outgoing bus"). In Figure 103, a curve (taken from Schwenkhagen[63]) for two parallel busses has been added.

[61] F. Walter, *Elektrotech. Z.*, **53**, 840 (1932).

[62] C. Danatt and S. W. Redfearn, *World Power*, **14**, 397, 492 (1930).

[63] H. Schwenkhagen, *Arch. Elektrotech.*, **17**, 537 (1927).

(b) Round Busses

For round busses, more data are available. Figure 104, taken from Dwight,[64] plots the root of the ratio, frequency: d-c resistance (ohm per 1000 ft), *vs.* σ_α (or $R_{a\text{-}c}/R_{d\text{-}c}$). The increase is larger for tubes with a wall thickness, *t*, which is high compared with the diameter, *d*. The

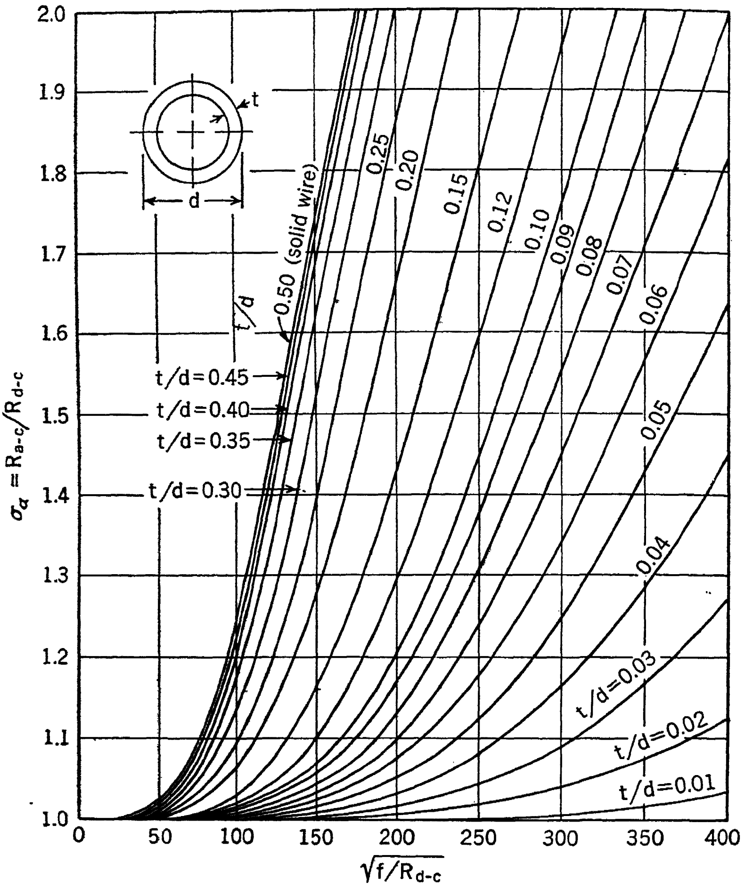


FIG. 104.—Resistance increase for tubes and rods.[64]

strong influence of frequency on losses can easily be read from the figure. For instance, with 4000 ohms/1000 ft d-c resistance, the curve for $2\,t/d$ is 0.3, and $\sigma_\alpha = 1.65$ as the ordinate. If the frequency is reduced to approximately 40% (25 instead of 60), the abscissa becomes $\sqrt{25/60} \times 4000 = 2580$. For an abscissa of 2580 and a value for $2\,t/d$ of 0.3, the ordinate is 1.15.

[64] H. D. Dwight, *Trans. Am. Inst. Elec. Engrs.*, 41, 203 (1922).

The proximity effect has also been studied by Dwight.[64] He suggests introducing a proximity effect ratio, σ_p, by which σ_α must be multiplied in order to allow for the proximity effect. Figure 105 is taken from the same paper. The same characteristic figure (root of the ratio, frequency: d-c resistance) is plotted *vs.* the correction factor, σ_p. Three curves are given holding for a distance from the center line of one bus to the center line of the other bus of 1.0, 1.5, and 2.0 times the diameter. For the latter distance, the proximity effect ratio, σ_p, does not exceed 1.13. For greater distances, the proximity effect can be neglected.

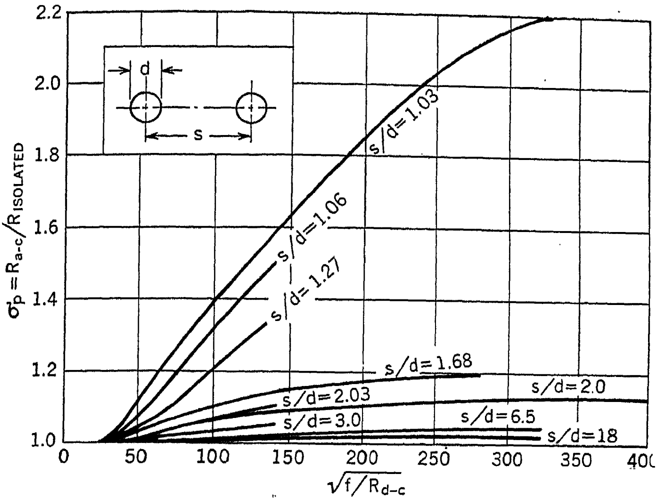


FIG. 105.—Proximity effect of round busses.[64]

The total a-c resistance of round conductors—solid or hollow—has been studied by Arnold.[65] Because of the proximity effect, the resistance depends on the geometrical arrangement of the busses. Arnold gives the results for the following cases: single-phase system (two equal conductors); three-phase system (three equal conductors in one plane); three-phase system (three equal conductors in triangular arrangement). Let the ratio, a-c resistance: d-c resistance, for an isolated conductor be $\sigma_{\alpha I}$, whose value can be taken from Figure 104. The values of σ_α for the three cases mentioned above are calculated by multiplying $\sigma_{\alpha I}$ by a correction factor, which is in all cases a function of the geometric arrangement, the frequency, and the electric d-c conductivity. Let d represent the outside diameter of the conductor, t the wall thickness, s the spacing (all in cm), and f the frequency in cycles per second, and the electric conductivity of the conductor in cgs units be expressed as $\lambda = 1/$resis-

[65] A. H. M. Arnold, *J. Inst. Elec. Engrs. London,* **78,** 580 (1936); **79,** 595 (1936).

tivity (ohm $\times$ cm $\times$ 10^9). The following relationships also are involved:

$$\alpha = d/s$$
$$\beta = 2\,t/d$$
$$z = 79\,t^2 f\lambda \text{ (for copper at 20 C, } z = 0.0458\,t^2 f)$$
$$\gamma = 1 - \beta(1 + \tfrac{1}{4}z^2) + 10\,\beta^2/(20 + z^2)$$
$$\phi = \frac{z^2(2-\beta)^2}{z^2(2-\beta)^2 + 16\,\beta^2}$$
$$\mu = \gamma\phi\alpha^2$$

For a single-phase system with two equal conductors (Fig. 106*a*) the value of the correction factor is given by Arnold as follows:

$$\sigma_{\alpha II} = \sigma_{\alpha I}(1-\mu)^{-\frac{1}{2}} \tag{36}$$

For a three-phase system (three equal conductors in one ~~phase~~ plane—Fig. 106*b*) the resistance of the inside and the outside resistors is different:

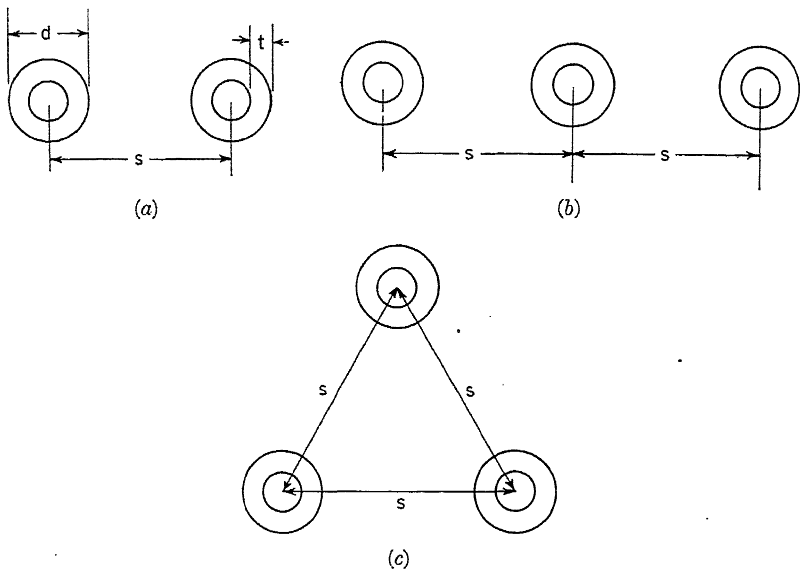


FIG. 106.—Bus arrangement: *a*, single-phase system; *b*, three-phase system, all conductors in one plane; *c*, three-phase system, conductors arranged in triangle.

the middle conductor has a skin effect factor, $\sigma_{\alpha IIIi}$; the outside conductors have a skin effect factor, $\sigma_{\alpha IIIo}$; and the average value, $\sigma_{\alpha IIIa}$, is:

$$\sigma_{\alpha IIIa} = (2\,\sigma_{\alpha IIIo} + \sigma_{\alpha IIIi})/3 \tag{37}$$
$$\sigma_{\alpha IIIi} = \sigma_{\alpha I}(1-\mu)^{-\frac{1}{2}}(1+\mu) \tag{38}$$
$$\sigma_{\alpha IIIo} = \sigma_{\alpha I}(1-\mu)^{-\frac{1}{2}}(1-\tfrac{1}{8}\mu) \tag{39}$$
$$\sigma_{\alpha IIIa} = \sigma_{\alpha I}(1-\mu)^{-\frac{1}{2}}(1+\tfrac{1}{4}\mu) \tag{40}$$

For a three-phase system having the phases arranged in triangle (Fig. 106c) the following formula has been given by Costello in the discussion of the paper of Arnold:

$$\sigma_{\alpha IIIT} = \sigma_{\alpha I}(1 - \mu)^{-\frac{1}{3}}(1 + \tfrac{1}{4}\,\mu - \tfrac{1}{4}\,\mu^3 - \tfrac{1}{3}\,\mu^9) \tag{41}$$

Since calculation of the values of $\sigma_{\alpha II}$ and $\sigma_{\alpha III}$ (Eqs. 36, and 38 to 41) would be quite cumbersome, graphs have been developed to simplify their use. Figure 107, an alignment chart, permits the reading of z for

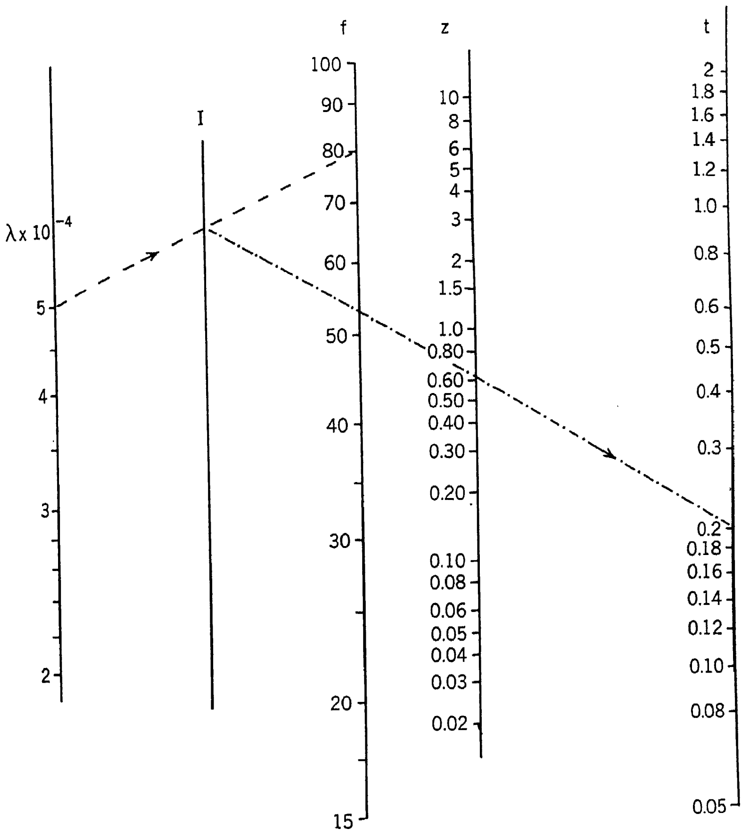


Fig. 107.—Alignment chart for z. $z = 79\,t^2 f/\lambda$.

any value of t, f, and λ. To use the chart, draw a line (arrow is an example) from λ to f, note the intersection with I, connect the intersection (dash-dot arrow) with the correct point on t. The intersection with z gives the value of z.

Figure 108 is a combined graph and alignment chart. On the left side, a graph of $\phi\gamma$ *vs.* z is plotted for different values of β; in the center

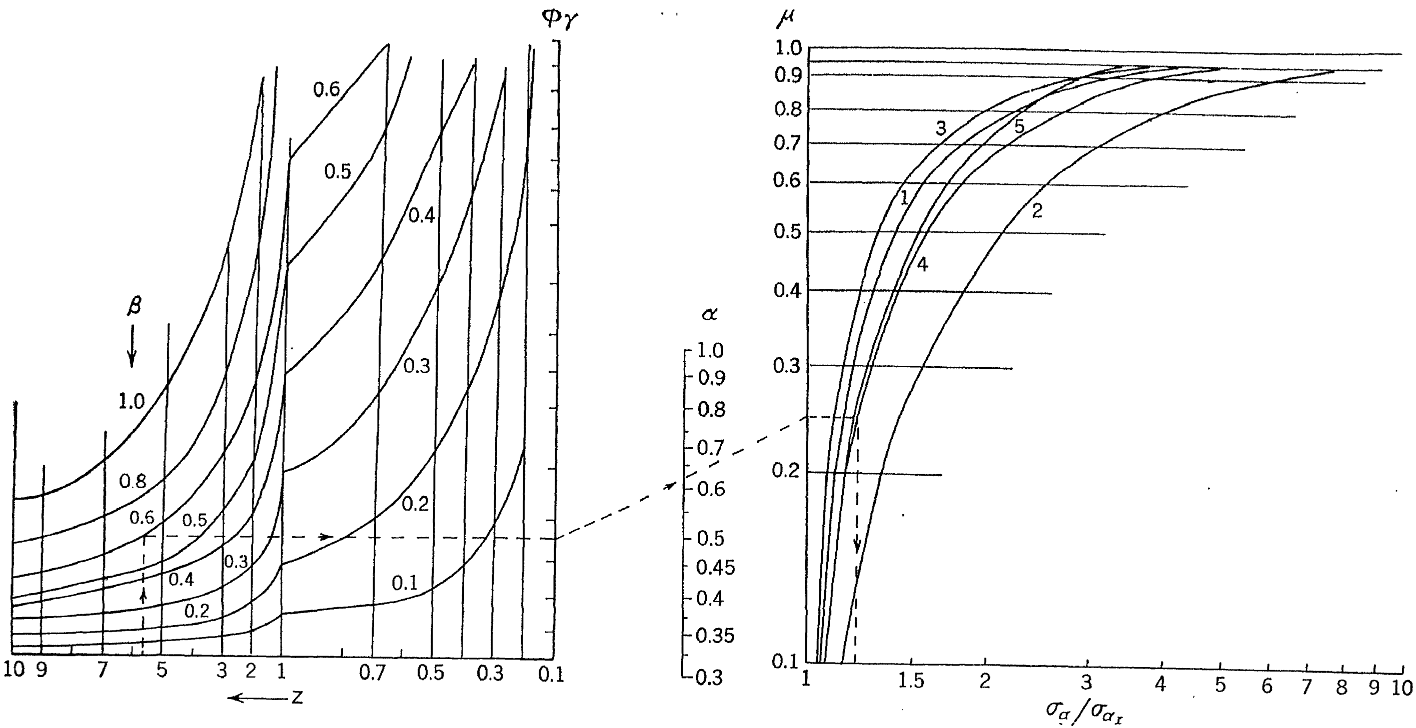


FIG. 108.—Skin effect factors. Evaluation of equations (36) to (41).

an alignment chart combines $\phi\gamma$, α, and μ; and the right side shows the fractions $\sigma_{\alpha II}/\sigma_{\alpha I}$ and $\sigma_{\alpha III}/\sigma_{\alpha I}$ for different values of μ. In order to use the chart, start on the left side with the known value of z (see Fig. 107), draw a vertical line to the appropriate value of β, and thence a horizontal to $\phi\gamma$; from the point thus found on the $\phi\gamma$ axis, draw a line through the value of α to the μ axis, and from this point, a horizontal to the curve showing σ_α (*i. e.*, $\sigma_{\alpha II}$, $\sigma_{\alpha IIIa}$, etc.); from there, drop a perpendicular to the abscissas and read the multiplication factor $\sigma_\alpha/\sigma_{\alpha I}$. The line is marked $\sigma_\alpha/\sigma_{\alpha I}$ (not $\sigma_{\alpha II}/\sigma_{\alpha I}$, $\sigma_{\alpha IIIT}/\sigma_{\alpha I}$, etc.) because it shows the ratio of any $\sigma_\alpha/\sigma_{\alpha I}$. An example in dotted lines illustrates the procedure.

(*c*) *Cables* (*Stranded Wire*)

The skin effect in stranded cables is different from that in solid wires, one reason being that the d-c resistance per unit length is different. Kennelly and Affel [66] show that, for a seven-wire cable, the spiraling is not of great influence. Even for the values, $\sqrt{f/R_{\text{d-c}}} = 225$ ($R_{\text{d-c}} = 0.0012$ for $f = 60$), and the most unfavorable conditions of spiraling (pitch of spirals per diameter of cable = 8), the resistance as compared with the unspiraled cable increases only about 7.5%.

For furnace calculations involving stranded cables, it is probably safe to work with the a-c resistance obtained for round conductors from Figure 104.

2. Reactance

The reactance of bus lines has been the subject of a number of detailed studies. An excellent general review of what has been done has been published by Rosa and Grover,[67] who compiled the most complete collection of formulas known at the time of publication (1911). An excellent bibliography on this problem is included. A paper by Levy [68] dealing specifically with the applications of such calculations to the design of arc furnaces may serve as a guide to further considerations.

In all calculations of self- and mutual induction, a characteristic figure, the "geometric mean distance," is used. The geometric mean distance is the nth root of the product of n distances between all the various pairs of points in the line, n being infinity. For further explanation, refer to Levy.[68] The geometric mean distance of a rectangular section from itself is:

$$R = 0.2235\,(a + b)$$

[66] A. E. Kennelly and H. A. Affel, *Proc. Inst. Radio Engrs.*, 4, 523 (1916).

[67] E. B. Rosa and F. W. Grover, *Natl. Bur. Standards Sci. Technol. Papers*, 5, No. 169 (1913).

[68] C. C. Levy, *Trans. Am. Inst. Elec. Engrs.*, 51, 903 (1932).

where a and b are length and breadth of the rectangle measured in centimeters (ln *standing for logarithm to the base e and* $\tan^{-1}$ *designating arc tangent*).

The geometric mean distance of rectangular sections from one another is found from:

$$\ln R = \frac{d^2}{b^2} \ln d + \tfrac{1}{2}\left(1 - \frac{d^2}{b^2}\right) \ln (d^2 + b^2) + 2\frac{d}{b} \tan^{-1} \frac{b}{d} - \frac{3}{2} \qquad (42)$$

where b is the breadth of the rectangle and d is the distance from center to center. The geometric mean distance of a circular area (of radius r) from itself is:

$$R = 0.779\, r$$

The self-inductances and mutual inductances can now be found from the following formulas (all geometric dimensions in cm; for self-inductance of a round conductor return lead at practically infinite distance, s = length, r = radius of conductor):

$$L = 2\,s\left(\ln \frac{2\,s}{r} - \frac{3}{4}\right) \qquad (43)$$

For mutual inductance of two round conductors:

$$M = 2\,s\left(\ln \frac{2\,s}{d} - 1 + \frac{d}{s}\right) \qquad (44)$$

For self-inductance of a straight rectangular bar:

$$L = 2\,s\left(\ln \frac{2\,s}{R} - 1 + \frac{R}{s}\right) \qquad (45)$$

For mutual inductance of two parallel rectangular straps:

$$M = 2\,s\left(\ln \frac{2\,s}{R} - 1 + \frac{R}{s}\right) \qquad (46)$$

In this equation, R is the geometric mean distance between the rectangular conductors. R can be read from Figure 109, the four parts of the figure covering four ranges.

To facilitate the use of Equations (43) to (46), refer to Figure 110, in which two curves are shown, Curve a representing Equation (43) and Curve b representing Equations (44) to (46), which are mathematically identical. Curve b shows a minimum.

Dwight [69] has determined the mutual inductance for pairs of stranded cables. If d designates the outside diameter of each cable and s their distance from center to center (both values measured in consistent units),

[69] H. D. Dwight, *Elec. World*, 61, 828 (1913).

then the inductance (in h per ft) is:

$$L = 0.14036 \log \frac{ms}{d} \times 10^{-6} \tag{47}$$

factor m depending on the number of strands in the cable. It can be read from Figure 111.

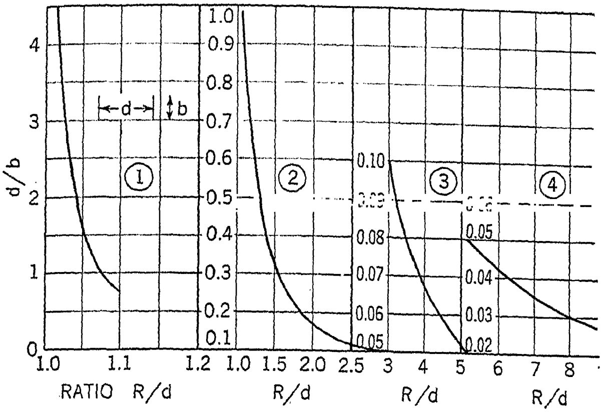


FIG. 109.—Geometric mean distance between thin straps. The four parts of the chart cover four ranges respectively.[68]

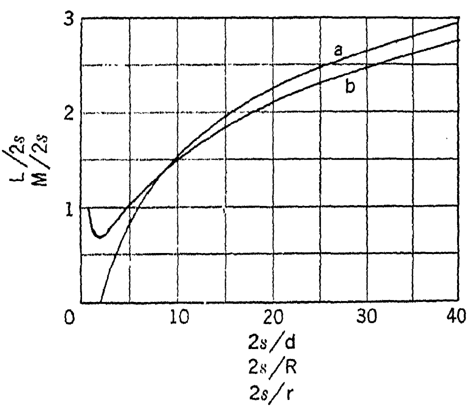


FIG. 110.—Self- and mutual inductance of round and rectangular conductors. Curve a, round conductor; curve b, rectangular conductor.

For multiple conductors, two solutions are possible. The actual solution calls for determining the mutual inductance of each conductor with respect to every other conductor, as well as the self-inductance, and

then taking the average of all these values. This procedure is necessary in the case of interlaced conductors.

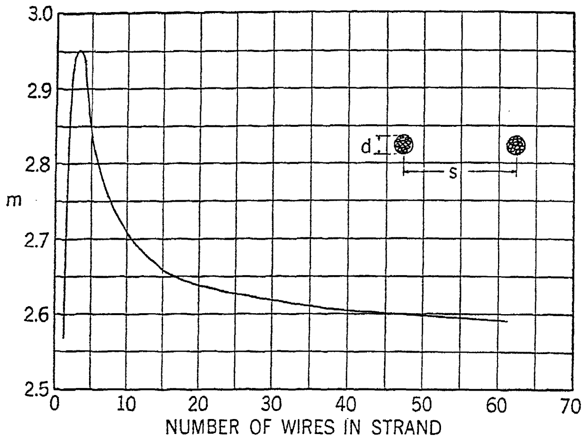


FIG. 111.—Factor m for mutual inductance of standard cables (after Dwight).

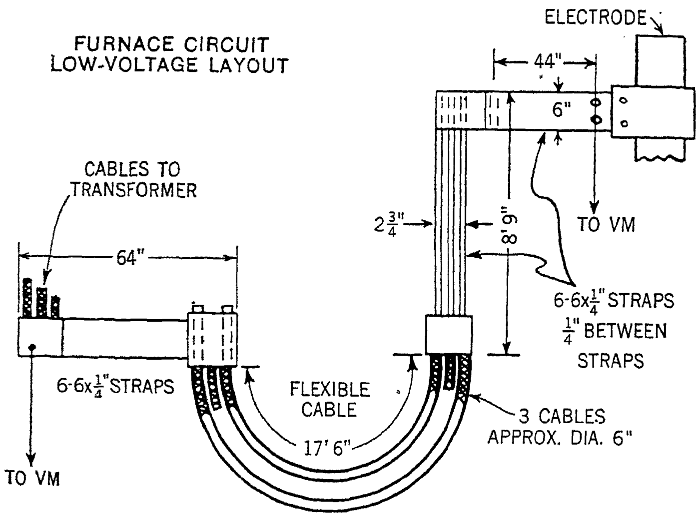


FIG. 112.—Arrangement of busses for furnace.[68]

The following example of calculating the inductance of a bus system is taken from the article [68] by Levy.[70]

For this example a furnace with delta-connected secondary was chosen, the low-voltage leads being arranged as shown in Figure 112. In this example

[70] Acknowledgment is made to C. C. Levy and the *American Institute of Electrical Engineers* for permission to reproduce this example.

the portion of the low-voltage bus across which impedance drop was measured does not include any interlaced connections.

Section 1—Horizontal run on top of furnace (straight portion).

Section 2—Horizontal run on top of furnace (curved portion).

Section 3—Vertical run on furnace to point at which flexibles are connected.

Section 4—Flexible connections.

Section 5—Horizontal run from flexible leads to point at which transformer cables are connected.

Section 1.—For arrangement of straps see Figure 113.

$$\text{Length} = 44 \times 2.54 \text{ cm} = 112 \text{ cm}$$

Self-inductance phase A:

$$L = 2s\left(\log\frac{2s}{R} - 1 + \frac{R}{s}\right) = 224 \times 2.8553 = 640 \text{ cm}$$

$$R = .223\,(6 + 2.75) \times 2.54 \text{ cm} = 4.95 \text{ cm}$$

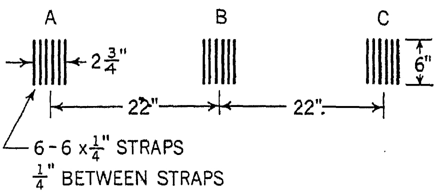


FIG. 113.—Arrangement of straps on furnace (section I of circuit).[68]

For mutual of A from B we have 22-in separation:

$$d/b = 22/6 = 3.66;\ \text{from curve } 1,\ \text{Fig. 109}\ R/d = 1.02;$$
$$R = d \times 1.02 \times 2.54 = 57 \text{ cm}$$

$$M = 224\left(\log\frac{224}{57} - 1 + \frac{57}{112}\right) = 224 \times 0.8861 = 198 \text{ cm}$$

For mutual from A to C 44-in separation:

$$d/b = 44/6 = 7.33;\ \text{therefore } R = 44 \times 2.54 = 112 \text{ cm}$$

$$M = 224\left[\log\frac{224}{112} - 1 + \frac{112}{112}\right] = 224 \times 0.693 = 155 \text{ cm}$$

Total inductance phase A for Section 1:

$$\begin{aligned}
\text{Self-inductance} &= 640 + j0 \\
\text{Mutual from } A \text{ to } B &= -\ 99 + j172 \\
\text{Mutual from } A \text{ to } C &= \underline{-\ 78 - j134} \\
& \quad\ 463 + j38
\end{aligned}$$

Section 2 is a continuation of Section 1 but with variable spacing between busses.

$$\text{Initial separation} = 22 \text{ in, final separation} = 10 \text{ in,}$$

$$\text{equivalent separation} = \sqrt{10 \times 22} = 14.8 \text{ in}$$

$$\text{Length} = 24 \text{ in} = 61 \text{ cm}$$

$$L = 122 \left[\log \frac{122}{4.95} - 1 + \frac{4.95}{61}\right] = 122 \times 2.2843 = 278 \text{ cm}$$

For M, from A to B separation = 14.8 inches

$$d/b = 14.8/6 = 2.46; \; R/d = 1.04; \; R = 1.04 \times 14.8 \times 2.54 = 39 \text{ cm}$$

$$M = 122 \left[\log \frac{122}{39} - 1 + \frac{39}{61}\right] = 122 \times 0.7842 = 96 \text{ cm}$$

For M, from A to C separation = 29.6 in:

$$R = 29.6 \times 2.54 = 75 \text{ cm}$$

$$M = 122 \left[\log \frac{122}{75} - 1 + \frac{75}{61}\right] = 122 \times 0.7186 = 88 \text{ cm}$$

Total inductance phase A for Section 2:

Self-inductance	=	$278 + j0$
Mutual A from B		$-\ 48 + j83$
Mutual A from C		$-\ 44 - j76$
		$186 + j7$

Section 3.—Vertical run on furnace. Six $6'' \times \frac{1}{4}''$ straps on 10-in centers:

$$\text{Length } s = 105 \text{ in} = 266 \text{ cm}$$ - 266 cm

$$L = 2s \left[\log \frac{2s}{R} - 1 + \frac{R}{s}\right]$$

$$R = .223 \times (6 + 2.75) \times 2.54$$

$$L = 532 \times 3.6915 = 1963 \text{ cm}$$

For mutual of phase A from phase B:

$$d/b = 10/6 = 1.6; \; R/d = 1.05; \; R = 1.05 \times 10 \times 2.54 = 26.8$$

$$M = 532 \left(\log \frac{532}{26.8} - 1 + \frac{26.8}{266}\right) = 532 \times 2.0867 = 1110 \text{ cm}$$

For M, from A to C:

$$d/b = 20/6 = 3.33; \; R/d = 1.02 \times 20 \times 2.54 = 52$$

$$M_{a\text{-}c} = 532 \left(\log \frac{532}{52} - 1 + \frac{52}{266}\right) = 532 \times 1.5254 = 810 \text{ cm}$$

Total inductance for phase A for Section 3:

$$
\begin{aligned}
\text{Self-inductance} &= 1963 + j0 \\
\text{Mutual from } A \text{ to } B &= -\ 555 + j960 \\
\text{Mutual from } A \text{ to } C &= -\ 405 - \cancel{j1700}\ j700 \\
\hline
&\ \ 1003 + j260
\end{aligned}
$$

Section 4.—Flexibles.

Length—17 ft 6 in = 210 in = 534 cm

Diameter = 6 in; spacing = 14 in

$R = 7.62$ cm

$$L = 1068\left[\log\frac{1068}{7.62} - \frac{3}{4}\right] = 4475 \text{ cm}$$

For mutual A from B, spacing is 14 in or 35.6 cm:

$$M = 1068\left[\log\frac{1068}{35.6} - 1 + \frac{35.6}{534}\right] = 1068 \times 2.4677 = 2640 \text{ cm}$$

For mutual of A from C, spacing = 71 cm:

$$M = 1068\left[\log\frac{1068}{71} - 1 + \frac{71}{534}\right] = 1068 \times 1.841 = 1960 \text{ cm}$$

$$
\begin{aligned}
\text{For phase } A\text{, total} &= 4475 + j0 \\
&\ -1320 + j2280 \\
&\ -\ \ 980 - j1700 \\
\hline
&\ \ \ 2175 + j580
\end{aligned}
$$

Section 5.

Length = 64 in = 163 cm

Spacing between phases = 14 in

Each phase, six straps 6″ × ¼″ with ¼-in spacing

Equivalent conductor cross section = 6″ × 2¾″

$R = .223 \times (6 + 2.75) \times 2.54 = 4.95$ cm

$$L = 326\left[\log\frac{326}{4.95} - 1 + \frac{4.95}{163}\right] = 326 \times 3.22 = 1050 \text{ cm}$$

For M, from A to B, we have:

$d/b = 14/6 = 2.33$; $R/d = 1.04$; $R = 1.04 \times 14 \times 2.54 \text{ cm} = 37$ cm

$M = 326 \times 1.414 = 460$ cm

For M, from A to C, we have:

$$d/b = 28/6 = 4.66; \quad R/d = 1.015; \quad R = 28 \times 1.015 \times 2.54 = 72 \text{ cm}$$

$$M = 326 \times 0.9681 = 315 \text{ cm}$$

Total inductance phase A for Section 5:

Self-inductance	=	$1050 + j0$
Mutual from A to B	=	$-230 + j400$
Mutual from A to C	=	$-158 - j272$
		$662 + j128$

TOTAL-INDUCTANCE FOR PHASE A

Section 1,	$463 + j38$
Section 2,	$186 + j7$
Section 3,	$1003 + j260$
Section 4,	$2175 + j580$
Section 5,	$662 + j128$
	$4489 + j1013$

$X_L = 2\pi f L \times 10^{-9} = 377 \times 4490 \times 10^{-9}$ ohm $= 0.00169$ ohm
Drop when carrying 8,400 amp = 14.19 v

TEST RESULTS

Average of eight readings on phase A = 4.2 amp
Current transformer ratio = 2000/1
Average secondary current = 8400 amp
V^m readings of impedance drop phase A
Average of eight readings = 13.9 v
Calculated inductive reactance = 0.00169
Calculated reactive drop = 14.19 v

This agrees with the measured impedance drop within reasonable limits of accuracy.

The approximate solution consists of taking the outside width of the group of conductors (including metal and space) and introducing their values in the equation for R. For the arrangement shown in Figure 114, $R = 0.2235 \times (6 + 2.75) \times 2.54$ in per cm.

For very large furnaces, Andreae[71] has published a set of tables giving mutual and self-inductances of bus systems of various sizes. The tables apply to rectangular three-phase furnaces with three electrodes in one row. Such arrangements are used mainly in ferro-alloy furnaces and

[71] F. V. Andreae, *Trans. Electrochem. Soc.*, 67, 150 (1935).

electrochemical units employing very heavy currents. Furnaces for the melting of steel generally have the electrodes arranged in triangles. Values of mutual and self-inductance are given for various combinations of conditions: distances between center lines of electrodes, different lengths of electrodes, electrode diameter, currents ranging from 18,000–36,000 amp. For detailed results (all of which apply to 60-cycle current), the tables should be consulted. The order of magnitude of inductive resistances are as follows:

Self-inductance of the outside phases (1 and 3 in Fig. 115):

18,000 amp, 1000 L, 0.8514 to 1.7989
24,000 amp, 1000 L, 0.9390 to 1.8199
36,000 amp, 1000 L, 0.9975 to 1.8196

Self-inductance of the inside phase (2 in Fig. 115):

18,000 amp, 1000 L, 0.6106 to 1.3285
24,000 amp, 1000 L, 0.6805 to 1.3384
36,000 amp, 1000 L, 0.7658 to 1.3749

Mutual inductance between phases 1 and 2 or 2 and 3:

18,000 amp, 1000 M_i, 0.1128 to 0.5280
24,000 amp, 1000 M_i, 0.1204 to 0.5043
36,000 amp, 1000 M_i, 0.1298 to 0.4869

Mutual inductance between phases 1 and 3:

18,000 amp, 1000 M_e, 0.0053 to 0.2754
24,000 amp, 1000 M_e, 0.0037 to 0.2495
36,000 amp, 1000 M_e, 0.0205 to 0.2434

The average inductance is:

$$1000\,L = \frac{2\,L_e + L_i - 2\,M_i - M_e}{3} \times 1000$$

18,000 amp, 1000 L, 0.6579 to 1.2957
24,000 amp, 1000 L, 0.7360 to 1.3282
36,000 amp, 1000 L, 0.7927 to 1.3459

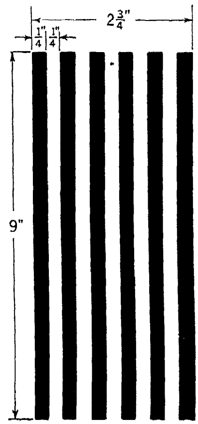


FIG. 114.—Determination of geometric mean distance for several parallel busses.

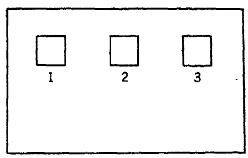


FIG. 115.—Numbering of electrodes.

V. TRANSFORMER AND REACTOR

The transformers used for arc furnaces are usually of the water-cooled, oil-immersed type. Their design differs in some respects from that of general power transformers. The currents are comparatively high and therefore the secondary winding is heavy. Because of the necessity for interweaving the busses, the parallel ends of the secondary are brought out unconnected (Fig. 116). In order to reduce the inductive loss, the connections from phase to phase are made as close to the

electrodes as possible. The secondary is therefore arranged in "open delta," *i. e.*, each phase is brought out with both ends. In the illustration, the reinforcements between the individual copper bars can be seen.

Today, the primary is almost always designed open so that the connection can be made in delta or Y. In addition, several taps are provided to give a larger number of steps. A total of six to eight different

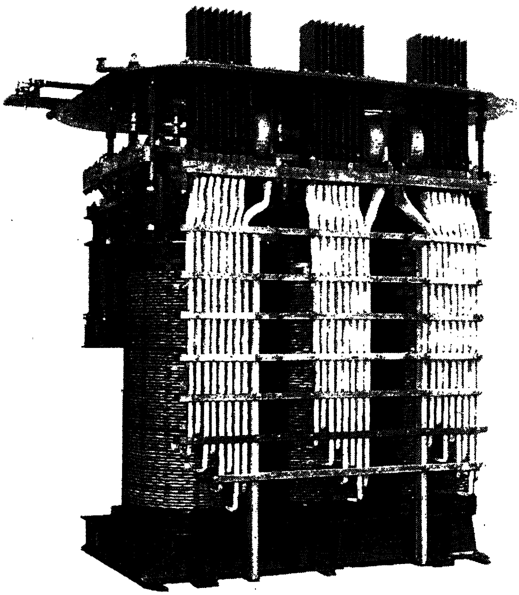


FIG. 116.—Oil-immersed, water-cooled transformer. (Courtesy *General Electric Co.*, Schenectady, N. Y.)

steps is considered the minimum necessary for flexible operation. Occasionally as many as 12 have been applied. Change of the primary connection from delta to Y (with unchanged secondary connections) lowers the secondary voltage in the ratio of 1:1.73. If four taps are provided, a total of eight voltages is available, because each tap can be used in either connection (delta or Y).

Because of the heavy short circuits with the resulting high mechanical forces, the primary winding must be held securely in position.

The switches used for changing taps and connections are of the oil-immersed type and may be either hand or motor operated. The changing of taps is usually carried out after switching off the load. The circuit

breakers are therefore operated very frequently. Whitehouse and Levy [72] state that as many as 130 interruptions per day have been observed.

The inherent reactance of furnace transformers is between 4 and 6%. This inherent reactance of small transformers (below approximately 5000 kva) is not sufficient to keep the surges within the usually desired limits. Small transformers are therefore frequently equipped with separate reactors which can be assembled in the oil tank of the transformer. They are of the iron-core type and are conveniently equipped with taps so that the inductance can be selected to meet the operational needs. These supplementary reactors range from 30% additional reactance (for small transformers) to 15% additional reactance (for transformers of 5000 kva).

The switch for selecting the reactor taps can be combined with the tap selector switch on the transformer. The latter also takes care of the Y-delta connection on the transformer. The reactor, instead of being ahead of the transformer, can be combined with the primary winding, in which case the reactor has two windings: one for line-to-line operation (delta) and the other for line-to-ground (Y).

VI. FURNACE CONTROL

It would be desirable to base the operation of a melting furnace on the temperature, which should be as high as possible without endangering the refractories or the metal itself. To maintain a given temperature, the rate of energy input would have to be changed continuously as the load and walls become saturated with heat, the control being so set that the temperature is kept at the maximum value safe for refractories and metal. This ideal method of control is impossible for two reasons:

(1) The temperatures at various points in the furnace are different at any given time (*e. g.*, close to the arc or near the wall).

(2) No instruments for industrial use are available which permit continuous measurement of the high temperatures involved.

The control of temperature is therefore replaced by control of input, the input being selected by the operator to meet (as well as he can judge) the requirements of operation.

Because of arc irregularities, it is usually desirable to start a cold charge with relatively low power. After melting down a pool under the electrodes, the arc burns more regularly. Then, in order to increase the speed of melting, the maximum available power is used. When the entire batch is melted, the power must be reduced or the temperature, especially that of the roof, would rise unduly. These various power requirements can be met by two means, applied either individually or

[72] Z. R. Whitehouse and C. C. Levy, *Iron Steel Engr.*, 48, 51 (1941). See also N. R. Stansel, *Gen. Elec. Rev.*, 40, 246 (1937).

combined: by a change of voltage and/or a change of current. Selection of the voltage is carried out by changing the taps on the transformer and the reactor; the latter influences the furnace voltage by changing the voltage drop in transformer busses and reactors. However, with a given voltage, the power—in a circuit containing an arc—remains unchanged only as long as the resistance of the arc remains unchanged; and the resistance of the arc is proportional to its length. It is thus essential to maintain the length of the arc constant. Although, in very small furnaces, this is performed by hand, lifting and lowering the electrodes generally is controlled automatically. Electrodes travel at a speed of approximately 2 ft per min.

A. Elements Controlling Electrode Movement

Any property which is in direct relationship to the length of the arc is suitable as a basis of control: resistance, length, power factor, power, current, electrode voltage. For practical reasons, most of these must be ruled out. The impracticability of using the arc resistance or its length or voltage as a basis of control is obvious. The power factor would be easily enough accessible to measurement but the changes of the factor within the usual range of operation are so small that very costly and sensitive equipment would be necessary. Thus only two possibilities remain, power or current. The former, in the neighborhood of maximum input into the arc ($W_{U_{max}}$, see page 171), does not change rapidly within the range of operation and therefore is also generally not used as a basis for electrode control. This leaves only the current as the usually accepted method of controlling the length. Of course the current alone does not determine the power; the setting of the current, which is to be held constant, must be changed as the melting proceeds and as transformer and reactor taps are changed.

B. Methods of Electrode Control

The measuring element in any electrode control based on maintaining constant current is a current relay, which is essentially an ammeter with or without contacts and generally without a scale. In the United States, in most cases only electrical means are used for transferring the impulse from this measuring device to the electrode moving devices. In Europe, hydraulic arrangements are sometimes used instead of electric ones. Hydraulic controls have also been used occasionally in the United States in recent years; all of them were designed exclusively for intermittent action. Now, however, controls with continuous action are available.

Figure 117 illustrates the wiring diagram of the Westinghouse control, showing the wiring of a control circuit for *one* electrode. For a

three-phase furnace, three such circuits are necessary. The bus feeding the (left) electrode carries a current transformer which feeds into a current relay (A). The a-c solenoid in A pulls up its core at a given current. The coil of A is shunted by resistor C. Thus A can be made to receive the same current with different electrode currents by adjusting C. The

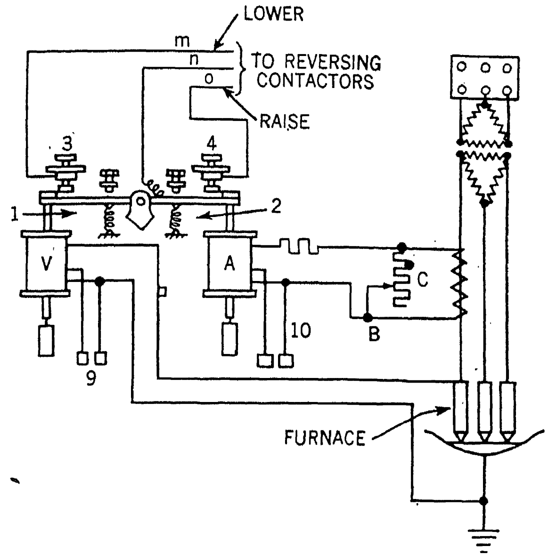


FIG. 117.—Control element circuit. (Courtesy *Westinghouse Electric and Manufacturing Co.*)

core of A and the core of a voltage relay, V, pull on the two sides of a lever, which is held in horizontal position by two springs, 1 and 2. The voltage relay acts on the electrode-to-ground voltage. If the lining is not sufficiently conducting, special connections between the shell and the bath must be made. The lever carries two contacts, 3 and 4; if the pull from the current and voltage coils are the same, both contacts are open; if the pull of the current coil is stronger, contact 4 closes and causes raising of the electrode. The length of the arc increases, the resistance increases, the current decreases, and the voltage increases. Thus the lever will, within a short time, be turned horizontal. The electrode will come to a standstill. If, however, the voltage coil pulls stronger, contact 3 will be closed and the electrode will be lowered with inverse results.

Figure 118 represents the wiring diagram for the electrode motor. Two double-throw contactors (R, L) are used. Their coils have one pole in common, which is connected to one pole of the power supply (PS). The other pole of the power supply is connected to n (Fig. 117). The other ends of the coils are connected to m and o, respectively (Fig.

117). The electrode control motors are d-c operated. The field is constantly on the d-c line. By reversing the polarity on the armature, the direction of rotation is changed. Thus, if contact 7 on contactor L closes and 8 opens, the current through the armature flows as follows:

$$+ \rightarrow A \rightarrow 7 \rightarrow \text{armature} \rightarrow B \rightarrow 6 \rightarrow -.$$

If contactor R closes, L opens automatically. The lever in Figure 117 allows energization of only one of the two contactors, and the current through the armature is reversed. If contactors L and R are open, con-

(5) (4)

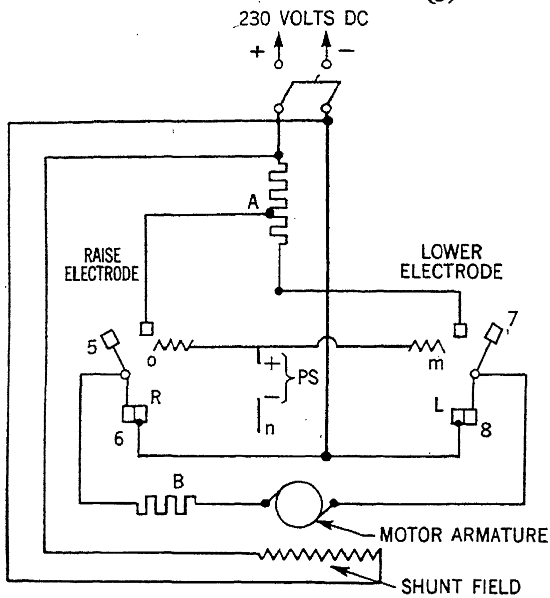


FIG. 118.—Electrode motor circuit. (Courtesy *Westinghouse Electric and Manufacturing Co.*)

tacts 6 and 8 are closed on the armature, which is short-circuited through resistor B. If the armature rotates, it acts as a generator with B as the load. It will therefore soon cease to rotate. Resistance A serves as starting resistance.

The two pairs of contacts, 9 and 10, shown in Figure 117 are for stabilization of control. They are attached to contactors L and R (Fig. 118): 9 closes simultaneously with 7 and 10 with 5, each shorting part of the respective coil (9 in V and 10 in A). Thus the hold of the lever is weakened and a tendency set up for the lever to return to a horizontal position very soon after it has made contact (either on 3 or 4). Consequently, the electrode motor, instead of running continuously, is turned

on and off repeatedly, giving the electrode current time to return to the desired value.

The G.E. system is similar. No voltage coil is used. The current coil is a d-c coil, fed through a copper oxide rectifier. Recently G.E. has applied the "Amplidyne" (Mohler [73]), Allis Chalmers the "Regulex," [74] and Westinghouse the "Rotatrol." [75] In all these types of control, the winch motors are not connected to a supply line but are each connected to separate generators, which run continuously and are excited

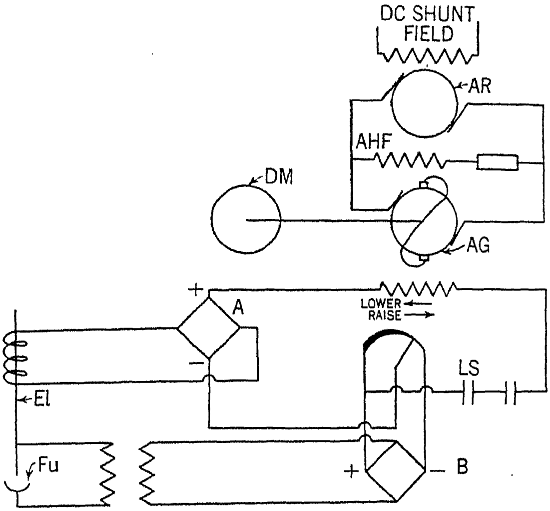


FIG. 119.—Amplidyne electrode control.

by direct current obtained by rectification from the respective electrode currents. The higher the electrode current, the higher is the voltage of the winch generator and the faster the motor will drive. As the current in each electrode approaches normal values, the voltage in its generator drops and its motor gradually stops.

The voltage of each generator depends not only on the current in its electrode but also on the voltage from this electrode to ground.

Allis Chalmers uses a standard generator, excited by a special "Regulex" exciter which connects the three generators, the three exciters and an a-c driven motor all on one shaft.

General Electric combines the generator and the exciter into one unit having one armature with several fields.

[73] F. Mohler, *Iron Steel Engr.*, **17**, 49 (1940).

[74] T. B. Montgomery, *Steel*, **111**, 145 (Sept., 1942); *Blast Furnace Steel Plant*, **30**, 861 (1942).

[75] R. A. Geiselman, C. C. Levy, and W. R. Harris, *Trans. Am. Inst. Elec. Engrs.*, **62**, 671, 983 (1943).

A schematic diagram of the G.E. unit is shown in Figure 119. The electrode motor is d-c operated and has a shunt field supplied directly from an independent source. The armature is fed from the Amplidyne generator, the polarity and voltage of which depend on the direction and the magnitude of the current in the "regulating" field. This regulating field is supplied by two rectifiers (*A* and *B*), which are connected through a rheostat with opposing polarity. Rectifier *A* supplies a direct current proportional to the electrode current, rectifier *B* a direct current proportional to the voltage electrode to ground. When the two voltages from *A* and *B* are equal, no current flows in the regulating field. As one or the other voltage predominates, the current in the regulating field will flow in the direction of dominant voltage. Limit switches are used to prevent overtravel of the electrodes. The rheostat is used to adjust the "equilibrium ratio" of *A* and *B* so that any desired electrode current can be maintained. All parts of secondary importance are omitted in the diagram, *e. g.*, means to adjust the voltage circuit for various arc voltages.

C. Hunting and Accuracy of Control

If the current in the electrodes deviates from the desired value and the control starts to operate, it tends to continue to operate and to "overshoot." The movement of the electrodes is reversed quite frequently; 20 reversals per minute are not considered excessive, even if no "hunting" occurs. Then a movement of the control in the opposite direction takes place and again the control may overshoot. This is known as "hunting" of the control. The danger of hunting is met by a number of means. The three more recent developments, Amplidyne, Regulex, and Rotatrol, in which the speed of electrode movement is made proportional to the deviation of current from the desired value, are probably the most important. In addition, the Amplidyne has a special "antihunt field" which decreases the effectiveness of the control field as the electrode current becomes smaller.

In the older methods with contactors dynamic breaking was used.

Besides these means, which are incorporated in the control, the furnace designer has others at hand to prevent hunting. The smaller the weight of the electrode, the less danger there is of hunting; in this respect, graphite electrodes have an important advantage over carbon electrodes. The use of counterweights for the electrodes also tends to decrease the inertia and the hunting. Finally, the use of ball bearings in the entire electrode movement and a careful system of greasing are necessary.

VII. OPERATING DIAGRAM AND EFFICIENCY

In previous sections, the various parts of arc furnaces and their design were discussed. In this section, we propose to show by means of operating diagrams the interdependence of the various parts of a furnace. It is of course the desire of users and designers alike to increase the output and, as far as possible, the efficiency of a furnace. In general, first interest is given to increase of output and only second thought to the efficiency. This is understandable from the fact that many other cost items besides energy cost are completely, or almost completely, independent of the output, and therefore their relative importance per ton of output decreases if the output increases. However, a careful study of this section will show that, in the case of arc furnaces, output is closely connected with efficiency. The output cannot be pushed beyond a certain limit if the efficiency is not increased. An apparent increase in input would only increase losses, and the useful energy would remain unchanged or even drop. From this section, it will become evident that careful consideration must be given to the thermal as well as to the electric losses of arc furnaces in order to increase output.

Arc furnaces have one characteristic which distinguishes them from all other electric furnaces and determines their behavior: the current and voltage in the arc (that is, at the point of transformation from electric energy to heat) are not proportional. This is attributable, first, to the current–voltage relationship of the arc proper and, second, to the fact that an important part of the energy is lost in the circuit outside the arc. The behavior of electric arcs can be studied by setting up their "characteristic," which is essentially a current–voltage diagram. The true characteristic of heavy-current arcs such as are used in furnaces has hardly been investigated. The following brief discussion of the general behavior of arcs will give a better understanding of the term "arc resistance" as used here.

In Figure 120, voltages are plotted *vs.* currents. An ordinary metallic resistor with a temperature coefficient of zero has a characteristic as shown by straight line b. The angle between the resistance line and the abscissa is ρ. Ctg $\rho = E/I$ is the resistance of the resistor (b). If resistor b were to have a different resistance, angle ρ_b would change. Doubling the voltage causes a 100% increase in current. Curve a_1 is the characteristic of an arc. The arc cannot exist below a minimum voltage, E_1, the magnitude of which is equal to the voltage drops in the arc ends. An increase in voltage automatically causes a decrease in current. One theory is that the arc characteristics are exactly hyperbolas and that the product of voltage $\times$ current—*i. e.*, the power in the arc proper—is always constant as long as its length is unchanged. If the

electrodes are brought closer together (shorter arc), a new characteristic, a_2, holds. For any given value of E_1, the value of $E \cdot I$ becomes smaller. The total power of the arc decreases: the shorter the length of an arc, the smaller is its power and its voltage for a given current. Changing the length of the arc is therefore "practically" the only way of changing its power. "Practically" is used because the value, E_1, depends to some extent on the diameter of the electrode.

Taking any one point, A_1, on the characteristic curve (*e. g.*, on a_1), the resistance of the arc at this point is $E_{B_1}/I_{B_1} = \text{ctg}\ \rho_{A_1}$. The resistance of the arc is now determined in a similar way for other points, A_2, A_3, etc. on a_1, and plotted on the ordinates of Figure 121. The

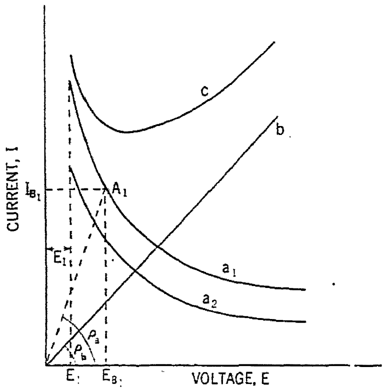


FIG. 120.—Arc characteristic.

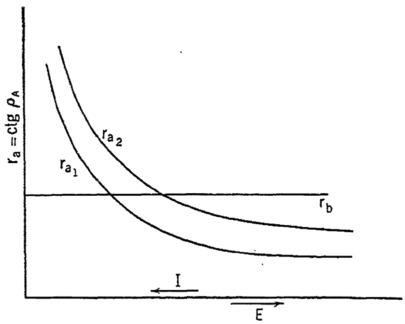


FIG. 121.—Arc resistance.

abscissa has two scales, one for E_1, the other for I. At small values of E, the resistance, r_{a_1}, is very large; it becomes smaller with increasing E, and reaches zero with infinitely high E. A similar curve, r_{a_2}, is obtained for a shorter length of the arc; however for any value of E the new resistance (for A_2) is higher. For the sake of comparison, the resistance line for a metallic resistor b is shown in Figure 121. It is a straight line, r_b, parallel to the abscissa axis.

Even if no external resistance is connected in series with the arc, the electrodes themselves form a series resistance. Thus, in nature, no arc without series resistance exists. If it could exist such an arc would be unstable because of its hyperbolic characteristic. If, in Figure 120, curves a and b are added, a third curve, c, is obtained. On the left part of curve c, a small decrease in voltage will result in an increase in current; on the right part, a small increase in voltage will result in a ~~decrease~~ in crease in current. Operation on the left side is not "stable." In order to secure a stable burning arc, a resistance (or inductance) must be connected in series to obtain a current–voltage characteristic of the type "c." ?

A. Conditions in a Noninductive Furnace (D-C Furnace)

For the sake of clarity, the inductive losses in the circuit will not e considered at first. The following section therefore holds only for d-c urnaces and serves as an introductory explanation.

With constant voltage E, a circuit essentially similar to that in Figure 122 esults, r_L being the resistance of all parts outside the arc—resistance in the usses and electrodes (r_L represents the resistance of the parts of the circuit ausing losses). For all discussions in his chapter, r_L and r_{LR} will be conidered constant. Actually, this value vill change with the change in length and in thickness as he electrodes burn off. Theoretically, t would therefore be necessary to carry ut separately the entire investigations or the lower and the upper limit of the lectrode resistance (and reactance). For ractical purposes, it is probably suficient to use mean values, the more o since the resistance of the electrodes s known only within a certain limit. The resistance of the arc, r_a, changes with the length of the arc and is therefore a variable resistance. The *current* n such a circuit is, of course:

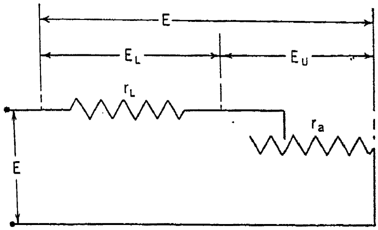


Fig. 122.—Circuit diagram of noninductive electrode furnace. E_U, useful voltage; r_a, arc resistance; r_L, resistance losses.

$$I = \frac{E}{r_L + r_a} \tag{48}$$

The total voltage drop, E, can be divided into two parts: one, E_L, covering the voltage drop in all "loss elements," r_L, and the other, E_U, covering the useful drop in the arc, r_a.

$$\frac{E_L}{E} = \frac{E_L}{E_L + E_U} = \frac{r_L}{r_L + r_a} \tag{49}$$

or:

$$E_U = \frac{r_a}{r_a + r_L} \cdot E \tag{50}$$

The *useful power*, W_U, is:

$$W_U = \frac{E_U^2}{r_a} \tag{51}$$

Substituting for E_U in Equation (51) from Equation (50) yields:

$$W_U = \frac{r_a}{(r_a + r_L)^2} \cdot E^2 \tag{51a}$$

By differentiating, the value of r_a can be found which gives the highest useful power, W_U: for $W_U = W_{U_{max}}$, r_a must equal r_L. The *total power*, W, in the circuit is:

$$W = \frac{E^2}{r_a + r_L} \tag{52}$$

Without any calculation it can be seen that the greatest power will be absorbed if $r_a = 0$.

$$W_{max} = \frac{E^2}{r_L} \tag{53}$$

For the condition, $W_U = W_{U_{max}}$, the total power in the circuit is:

$$W_{W_{U_{max}}} = \frac{E^2}{2r_L} \tag{54}$$

It is important that the highest total power, W_{max}, be different from, and in fact twice as high as, the total power at the point of *highest useful power* ($W_{W_{U_{max}}}$). Equation (51a) now reads for $W_U = W_{U_{max}}$:

$$W_{U_{max}} = \frac{r_L E^2}{4r_L^2} = \frac{E^2}{4r_L} \tag{55}$$

By comparing Equation (53) with Equation (54), it is readily seen that:

$$W_{U_{max}} = \tfrac{1}{2} W_{W_{U_{max}}} \tag{56}$$

This means that the highest output is obtained if the lost power is also $\frac{1}{2}W_{W_{U_{max}}}$ or, in other words, if the electric efficiency is only 50%.

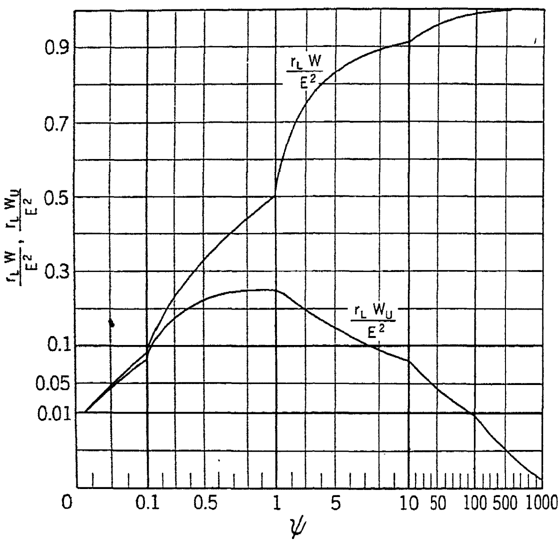


FIG. 123.—Characteristic curves for a noninductive furnace—W, W_u *vs.* ψ. Dark lines indicate change of scale.

From these equations, two very important facts can be learned: (1) The highest (electric) energy input in a d-c arc furnace for a given supply voltage is inversely proportional to the electric resistance of the busses and electrodes; and (2) the highest useful (electric) power input in a d-c arc furnace occurs when the electric efficiency is only 50%.

The *electric efficiency*, η (which in this chapter will always refer to the electric efficiency as defined by Eq. 57), can be expressed by the ratio:

$$\eta = W_U/W = \frac{E_U^2/r_a}{E^2/(r_a + r_L)}$$

By substituting for E_U from Equation (50):

$$\eta = \frac{r_a^2}{(r_a + r_L)^2} \cdot \frac{1}{r_a} \cdot (r_a + r_L) = \frac{r_a}{r_a + r_L} \tag{57}$$

By differentiating this equation, the value of η_{max} is found to be $\eta_{max} = 1$ for $r_a = \infty$. This means that an electric efficiency equal to "unity" can be

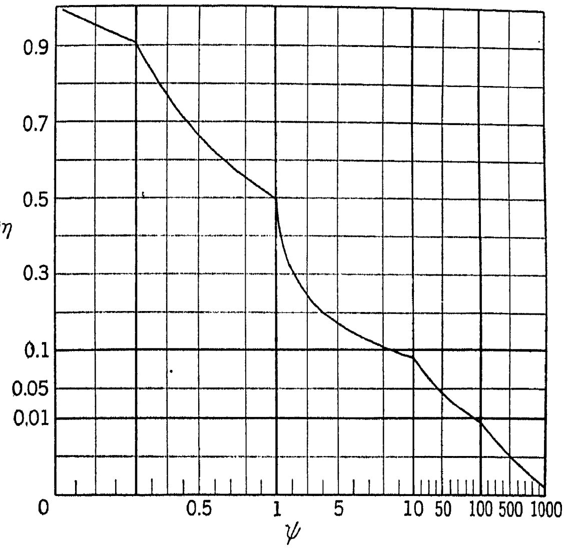


FIG. 124.—Characteristic curves for a noninductive furnace—η *vs.* ψ Dark lines indicate change of scale.

obtained if the useful resistance is infinite; but then the power input is of course zero. By expressing r_a as a multiple of r_L ($r_a/r_L = \psi$), η, r_LW/E^2, and r_LW_U/E^2 can be determined from Equations (57a), (52a), and (53a), respectively.

$$\frac{r_LW}{E^2} = \frac{1}{\psi + 1} \tag{52a}$$

$$\frac{r_LW_U}{E^2} = \frac{1}{\psi + 2 + \dfrac{1}{\psi}} = \frac{\psi}{(\psi + 1)^2} \tag{53a}$$

$$\eta = \frac{\psi}{\psi + 1} \tag{57a}$$

Figure 123 shows a plot of r_LW/E^2 and r_LW_U/E^2 (ordinates) and Figure 124 a plot of η *vs.* ψ. It is obvious that, if the electric efficiency becomes high,

approaching its maximum value, the useful power decreases steadily towards zero.

Three questions should be examined: At what value of η should a furnace be operated? What significance has the resistance, r_L? What significance has the furnace voltage, E? Before entering into a discussion of these questions, the assumption of a d-c operated furnace must be abandoned, an assumption made only because it greatly simplified conditions. Today, d-c furnaces are of course no longer in operation. The resistance, r_L, in an a-c furnace is no longer a purely ohmic resistance. The inductance of the circuit must be taken into consideration.

B. Single-Phase Furnaces with Inductance

In furnaces operating with alternating current, the basic circuit (Fig. 122) is replaced by the circuit shown in Figure 125. The inductance of the circuit is represented by $r_{Li} = 2\ \pi f L$ (see Eq. 37, page 64), where f is the frequency (in cycles per sec), L is the inductance (in h), and r_{Li} represents the resistance, inductive losses. In ordinary electrical terminology the inductive resistance is called inductance and is represented by x. For the reader not familiar with electrical terminology, the use of the words "inductive resistance," which is implied in "resistance, inductive losses," is more appropriate. It emphasizes the fact that the inductance (inductive resistance) in an a-c circuit has the same general result as a resistance, namely, to decrease the current at a given voltage. Inductive resistance is encountered in every part of the path of the current. The reactor (if any), the transformer in both its windings, the entire bus system, and finally the electrodes have inductive resistance. However, the arc resistance is purely ohmic.

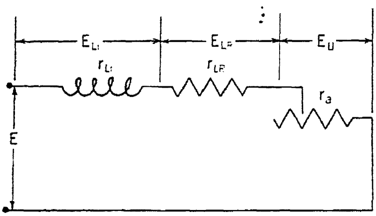


Fig. 125.—Basic circuit for a-c furnaces: r_L, inductive resistance (E_{Li}); r_{LR}, ohmic resistance (E_{LR}); r_a, arc resistance (E_U). The symbols in parenthesis indicate the corresponding voltage drops. Core losses of the transformer are not included. See page 182.

The circuit up to, but not including, the arc also has ohmic resistance, r_{Lr} (r_{Lr} representing resistance, ohmic losses). Again all parts of the current path contribute to the total ohmic resistance. Calculations of the inductive and ohmic resistances have been described above for the bus system (page 143); the values for the transformer are usually given by the manufacturer. Methods of measuring the resistances are discussed later on page 197. As mentioned previously, the voltage drops—

E_{Li} in r_{Li}, E_{LR} in r_{LR}, and E_U in r_a—cannot be added algebraically. The two ohmic drops (E_{LR} and E_U) are in phase. But E_{Li} is out of phase with these two, leading them by 90 degrees. The resistances must be added vectorially in order to determine the current. In Figure 125, the *current* (I) in the circuit is:

$$I = E/\sqrt{r^2_{Li} + (r_{LR} + r_a)^2} \tag{58}$$

The *useful power* is: $W_U = I^2 r_a$. Substituting for I from Equation (58):

$$W_U = E^2 \cdot \frac{r_a}{r^2_{Li} + (r_{LR} + r_a)^2} \tag{59}$$

By differentiation it can be found that $W_U = W_{U_{max}}$ for

$$r_a = \sqrt{r^2_{Li} + r^2_{LR}} \tag{60}$$

The expression, $\sqrt{r^2_{Li} + r^2_{LR}}$, is the *impedance* of the furnace in open circuit.

The condition for maximum useful power (Eq. 60) is very similar to that for the d-c furnace, where it has been found to be $r_a = r_L$, the impedance taking the place of the resistance. Introducing Equation (60) in Equation (59), the *maximum useful power* ($W_{U_{max}}$) is:

$$W_{U_{max}} = E^2 \cdot \frac{\sqrt{r^2_{Li} + r^2_{LR}}}{r^2_{Li} + (\sqrt{r^2_{Li} + r^2_{LR}} + r_{LR})^2} \tag{61}$$

The *total power* (W) in the circuit is:

$$W = I^2(r_{LR} + r_a)$$

Substituting again for I from Equation (58), W can be written:

$$W = E^2 \frac{r_{LR} + r_a}{r^2_{Li} + (r_{LR} + r_a)^2} \tag{62}$$

By differentiating it can readily be seen that the highest input, W_{max}, occurs if:

$$r_{Li} = r_{LR} + r_a \tag{63}$$

This equation will now be compared with Equation (60), showing the condition at which $W_{U_{max}}$ occurs. In introducing r_{Li} from Equation (63) into (62), the value for W_{max} can be found:

$$W_{max} = E^2 \frac{r_{Li}}{2\,r^2_{Li}} = \frac{E^2}{2\,r_{Li}} \tag{64}$$

For the maximum useful power, $W_{U_{max}}$, a total power, larger than the maximum useful power, $W_{WU_{max}}$, is required; this may be found by intro-

ducing into Equation (62) the value r_a from Equation (60):

$$W_{W_{U_{max}}} = E^2 \cdot \frac{r_{LR} + \sqrt{r_{Li}^2 + r_{LR}^2}}{r_{Li}^2 + (\sqrt{r_{Li}^2 + r_{LR}^2} + r_{LR})^2} \tag{65}$$

The *electric efficiency* is again (analogous to Eq. 57):

$$\eta = W_U/W = r_a/(r_{LR} + r_a) \tag{66}$$

By introducing the value for r_a from Equation (60) into (66), the efficiency at maximum useful power is found:

$$\eta_{W_{U_{max}}} = \frac{\sqrt{r_{Li}^2 + r_{LR}^2}}{r_{LR} + \sqrt{r_{Li}^2 + r_{LR}^2}} \tag{67}$$

It is noteworthy that the efficiency of the arc furnace does not directly depend on the inductive resistance, r_{Li}. However, r_{Li} is involved indirectly because r_a cannot be arbitrarily selected.

The *power factor*—$\cos\varphi$—of an arc furnace depends again on the load:

$$\cos\varphi = \frac{r_{LR} + r_a}{\sqrt{r_{Li}^2 + (r_{LR} + r_a)^2}} \tag{68}$$

The power factor at $W_U = W_{U_{max}}$ is found by introducing for r_a the value from Equation (60):

$$\cos\varphi_{W_{U_{max}}} = \frac{r_{LR} + \sqrt{r_{LR}^2 + r_{Li}^2}}{\sqrt{r_{Li}^2 + (\sqrt{r_{LR}^2 + r_{Li}^2} + r_{LR})^2}} \tag{69}$$

The power factor for $W = W_{max}$ is found by introducing for r_a the value from Equation (63):

$$\cos\varphi_{W_{max}} = 0.707$$

In order to obtain general equations, two ratios are needed, the inductance factor, *i. e.*, the tangent of the phase angle in open circuit:

$$\chi = r_{Li}/r_{LR} \tag{70}$$

and the equivalent to the expression in the d-c furnace:

$$\psi = r_a/r_{LR} \tag{71}$$

With these two ratios, the expressions for W_U/E^2, W/E^2, and η can be determined from Equations (59), (62), and (66) as follows:

$$\frac{W}{E^2} \cdot r_{LR} = \frac{\psi + 1}{\chi^2 + (1 + \psi)^2} \tag{62a}$$

$$\frac{W_U}{E^2} \cdot r_{LR} = \frac{\psi}{\chi^2 + (1 + \psi)^2} \tag{59a}$$

$$\eta = \frac{\psi}{\psi + 1} \tag{66a}$$

These equations are represented in Figures 126, 127, and 128; the values for η, $(W/E^2)(r_{LR})$, and $(W_U/E^2)(r_{LR})$ are plotted *vs.* ψ. Different curves are drawn for different values of χ.

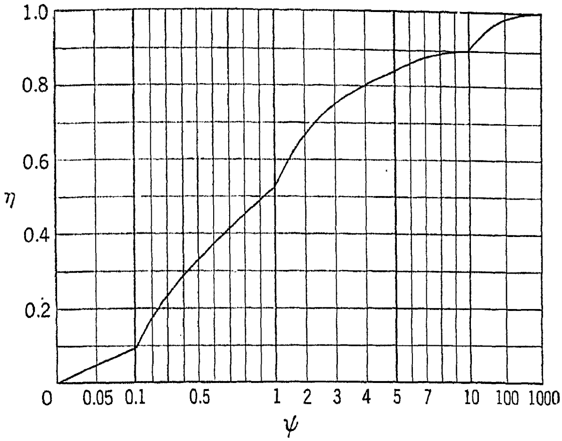


FIG. 126.—Electric efficiency expressed as function of ψ.

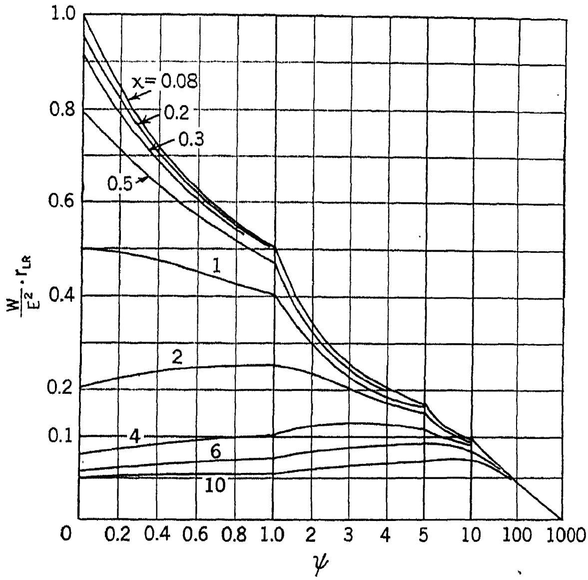


FIG. 127.—Total power expressed in dimensionless units as a function of ψ and χ.

It was found above that the useful power reaches its maximum if:

$$r_a = \sqrt{r_{Li}^2 + r_{LR}^2} \tag{60}$$

By dividing both sides by r_{LR}, this equation can be expressed by ψ and χ. The condition for $W_U = W_{U_{max}}$ is:

$$\psi = \sqrt{\chi^2 + 1} \tag{60a}$$

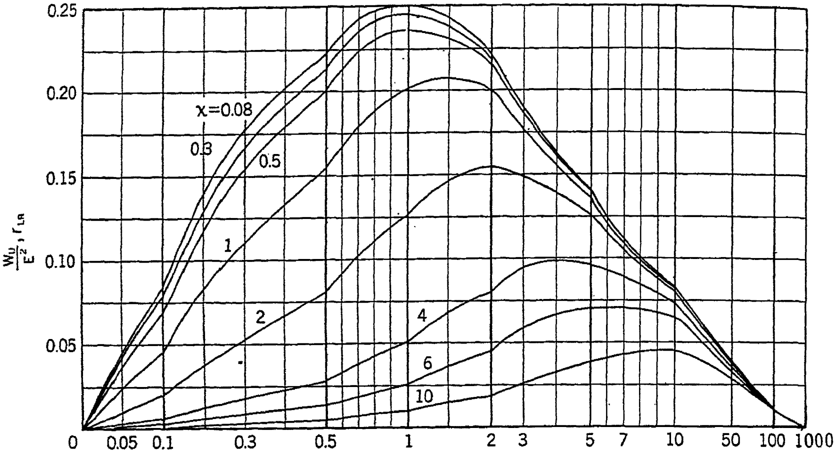


FIG. 128.—Useful power expressed in dimensionless units as a function of ψ and χ.

By introducing this value for ψ in Equations (59a), (62a), and (66a), the values of $W_{U_{max}}$, W at $W_{U_{max}}$ and η itself can be expressed as functions of ψ and χ:

$$\frac{W_{U_{max}}}{E^2} \cdot r_{LR} = \frac{\sqrt{\chi^2 + 1}}{\chi^2 + (1 + \sqrt{\chi^2 + 1})^2} = \frac{1}{2(\sqrt{\chi^2 + 1} + 1)} \tag{61a}$$

$$\frac{W_{W_{U_{max}}}}{E^2} \cdot r_{LR} = \frac{\sqrt{\chi^2 + 1} + 1}{\chi^2 + (1 + \sqrt{\chi^2 + 1})^2} = \frac{\sqrt{\chi^2 + 1} + 1}{2[(\chi^2 + 1) + \sqrt{\chi^2 + 1}]} \tag{65a}$$

$$\eta_{W_{U_{max}}} = \frac{\sqrt{\chi^2 + 1}}{1 + \sqrt{\chi^2 + 1}} \tag{67a}$$

It is very interesting to note that, in these equations, ψ does not appear. When χ becomes zero, the furnaces act like d-c furnaces and the equations yield values identical with those mentioned for this type of furnace.

Similarly, Equation (63), giving the conditions for maximum power input, W_{max}, can be expressed by ψ and χ. $W = W_{max}$ for:

$$\chi = \psi + 1 \tag{63a}$$

By again introducing this value in Equations (59a), (62a), and (66a), the values for η and W_U at W_{max} and the values of W_{max} itself can be

determined:

$$\frac{W_{max}}{E^2} \cdot r_{LR} = \frac{1}{2\chi} \tag{64a}$$

$$\frac{W_{U W_{max}}}{E^2} \cdot r_{LR} = \frac{\chi - 1}{2\chi^2} \tag{72}$$

$$\eta_{W_{max}} = \frac{\chi - 1}{\chi} \tag{73}$$

The total current as read on the ammeter can also be expressed by ψ and χ. From:

$$I = E/\sqrt{(r_{LR} + r_a)^2 + r_{Li}^2} \tag{58}$$

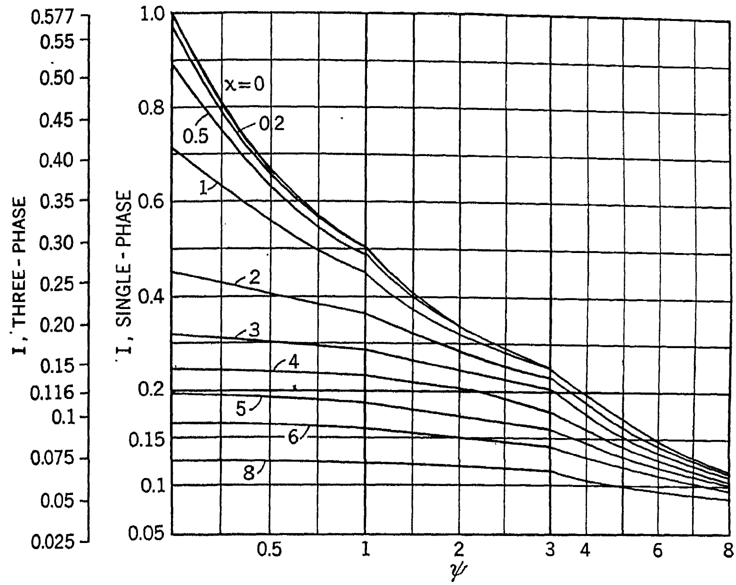


FIG. 129.—Current expressed in dimensionless units as a function of ψ and χ.

the value for Ir_{LR}/E can readily be determined; by introducing χ and ψ as above, the general expression for I can be found:

$$\frac{Ir_{LR}}{E} = \sqrt{\frac{1}{(1 + \psi)^2 + \chi^2}} \tag{58a}$$

In Figure 129, Ir_{LR}/E is plotted as a function of ψ and χ.

The value reached by I when $W_U = W_{U_{max}}$ is an important characteristic figure. It can be found by introducing Equation (60a) into Equation (58a):

$$\frac{I_{W_{U_{max}}} r_{LR}}{E} = \sqrt{\frac{1}{(1 + \sqrt{\chi^2 + 1})^2 + \chi^2}} = \sqrt{\frac{1}{2[(1 + \chi^2) + \sqrt{1 + \chi^2}]}} \tag{74}$$

Finally, the power factor can be expressed in general terms:

$$\cos \varphi = \frac{\psi + 1}{\sqrt{\chi^2 + (\psi + 1)^2}} \tag{68a}$$

$$\cos \varphi_{W_{L\,max}} = \frac{1 + \sqrt{1 + \chi^2}}{\sqrt{\chi^2 + (\sqrt{\chi^2 + 1} + 1)^2}} = \frac{1 + \sqrt{1 + \chi^2}}{\sqrt{2[(\chi^2 + 1) + \sqrt{\chi^2 + 1}]}} \tag{75}$$

Having now established the various equations, we can discuss them individually and the influence of the different factors can be analyzed. A summary of the more important equations will be found at the end of the section on three-phase furnaces (pages 188–189).

C. Discussion of the Equations

1. Current—Equations (58) and (58a)

From Equation (58a) and Figure 129 it is obvious that r_L and χ limit the current. If there is no load on the furnace, r_a and ψ become zero. From Equation (58) it is obvious that, if r_{Li} and r_{LR} become very small, I (for a given value of E) becomes exceedingly large. This is by no means a theoretical case; the resistance of the charge is practically negligible. Every time the electrodes hit the charge, the case of $r_a = 0$ occurs. As a consequence, the current reaches peaks which sometimes are very serious (see page 191). The necessity for limiting the current peaks calls for finite values of r_{Li} and r_{LR}. Inductive and ohmic resistances (r_{Li} and r_{LR}) are of course inherent in the different parts of the furnace (transformer, busses, electrodes). In some cases, the resistances must be artificially increased (by reactors) in order to limit the current.

For the purpose of limiting the current, it is of no importance if, in the denominator of Equation (58), r_{LR} and r_{Li} become large. Considerations of the efficiency call of course for high r_{Li} and low r_{LR}.

It is almost self-evident that (1) in order to obtain any useful power, the current must not be kept too low; and (2) the ratio of the maximum current (obtained in short circuit, $r_a = 0$) to the current at maximum load is a most important criterion and one of the legitimate starting points for furnace calculation (page 195). This ratio is only a function of χ: if $r_a = 0$, then $\psi = 0$ also. Equation (58a) becomes:

$$\frac{I_{max} r_{LR}}{E} = \sqrt{\frac{1}{1 + \chi^2}} \tag{76}$$

Dividing by Equation (74), both r_{LR} and E cancel out:

$$\frac{I_{max}}{I_{W_{L\,max}}} = \sqrt{2\left(1 + \frac{1}{\sqrt{1 + \chi^2}}\right)} \tag{77}$$

This equation is represented in Figure 130. The ratio $I_{max}/I_{W_{U_{max}}}$ is at least 1.414 ($= \sqrt{2}$), if $\chi = \infty$; the maximum it can reach is 2 (for $\chi = 0$).

2. Power—Equations (62), (62a), (64), (64a), and (65a)

The power absorbed by an arc furnace depends on both figures, χ and ψ. The smaller either of these two parameters, the higher the power becomes. With small values of ψ, χ becomes more important and, vice versa, with small values of χ, ψ becomes the deciding factor. The maximum power depends on r_{Li} only, not on r_{LR}. At first sight this appears strange. The explanation, however, is simple: maximum power is obtained if r_a assumes such a value that $r_{LR} + r_a = r_{Li}$. Hence, the maximum power is always the same, even if r_{LR} should change; a change in r_{LR} merely calls for a different value of r_a.

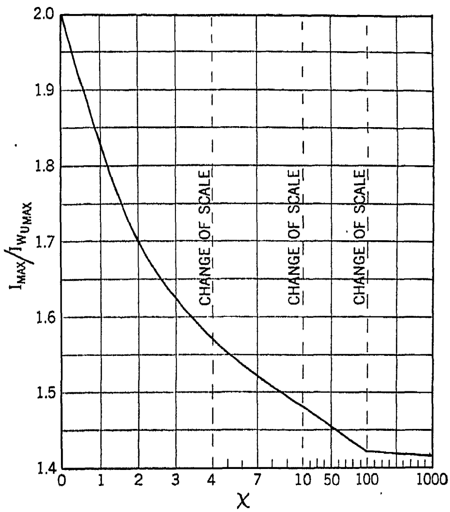


FIG. 130.—Ratio of maximum current to current at maximum useful power. Broken lines indicate change of scale.

In striving for the maximum output of the furnace, the operator aims at the maximum useful power input; this input is of course an important value and depends only on χ and r_{LR} (Eq. 65a). It is important to note that achievement of the maximum useful input cannot be determined by a wattmeter (see page 197). The smaller r_{LR}, the smaller will be the input for the maximum useful power.

3. Useful Power—Equations (59), (59a), (61), and (61a)

By comparing Equations (59a) and (62a) it is obvious that the useful power, W_U, is more influenced by ψ than is W. The maximum value

of W_U depends again on χ and r_{LR}. As r_{LR} becomes smaller, $W_{U_{max}}$ increases. The value of the latter is really one of the most important characteristic figures of the furnace, if not the most important. In a given furnace design, r_{Li} and r_{LR} are given, and therefore $W_{U_{max}}$ also (except for change in voltage or frequency, see pages 180 and 181).

The nature of the W_U curve is interesting: the maximum is always relatively flat. This point is useful in selecting the desired point of operation (Fig. 128).

$W_{U_{max}}$ is larger for small values of r_{Li} and r_{LR}. However, this causes a high value of the short-circuit current, and a compromise becomes necessary (see page 195). If the maximum useful power input is given, the only way to increase the output is by decreasing the heat losses, both direct and indirect, a matter of paramount importance. The usual opinion that efficiency "does not matter," only output, can no longer be upheld because *the thermal efficiency determines the output.* This will be more fully explained later (page 200).

4. Electric Efficiency—Equations (66), (66a), (67), and (67a)

Although the usefulness of an efficiency figure in characterizing a furnace is limited, in certain connections it is helpful. It should be kept in mind that electric efficiency decreases steadily with total furnace power. At zero input, efficiency is unity; it becomes zero at the largest input (no arc; short-circuit). Efficiency is not influenced by the inductive loss, *i. e.*, the efficiency curve plotted against ψ is the same for any value of χ. However, χ (as indication of the inductive resistance) determines the point on the η–ψ curve at which the furnace should be operated; and that is the point for $W_{U_{max}}$.

5. Power Factor—Equations (68), (68a), and (75)

The power factor at $\psi = \infty$ (open circuit) becomes unity (divide numerator and denominator in Eq. 68a by r_{LR}). At $\psi = 0$ (short circuit), the power factor, $\cos \varphi$, equals $1/\sqrt{\chi^2 + 1}$. The power factor for maximum useful power is always larger than that for maximum total power. Cos φ at the maximum total power is always, for any furnace, 0.707. Consequently, the power factor in every instance should be above 0.707—because the furnace should always be operated at maximum useful power. Inductive load and therefore poor power factor are the price paid for limitation of current surges. A compromise, which is discussed in detail on page 195, must be found.

D. Discussion of the Variables

1. Resistance Ratio

The "loss resistance" comprising the sum of ohmic resistances of a parts of the circuit except the arc is considered to be constant. Thi assumption is not accurate; temperature changes influence the resistivit of all parts—metallic as well as nonmetallic (electrodes). The contac resistance in the bus system may change. The greatest change o resistance, however, occurs in the electrode, partly because the lengtl varies (as the electrode burns off), partly as its shape changes by oxi dation of the cylindrical surface. For a given constant loss resistance however, the ratio, ψ, of the arc resistance to the loss resistance is a very convenient expression. The resistance of the arc, r_a, is changed by changing the length of the arc. However, the resistance is not in direc proportion to the length. A fixed amount is found in the "foot" of the arc (on the electrode and on the bath) and only the rest of the resistance is to some extent proportional to the length of the arc. The fixed values of the resistance at the foot are in turn different for different sizes and material of the electrodes and for different temperatures of the surroundings. Thus, the operator will in general not know the resistance of his arc—even if he should happen to know its length. An operating diagram based on the current of the furnace will, therefore, be shown later. Charts based on ψ can be used for any furnace whereas a diagram based on current (see page 185) applies to one given furnace only.

By way of information it may be mentioned that r_a may be of the order of magnitude of 0.003 ohm at normal operating conditions for 3000 kw per electrode.

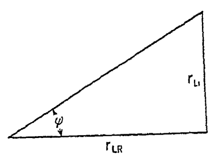


FIG. 131.—Phase angle in open circuit.

For a given furnace, ψ changes with the load and with the length of the arc. In one furnace any load smaller than the maximum can be obtained with different values of ψ, each different value corresponding to a different arc length. High ψ is obtained with high r_a and consequently with long length of the arc. If the furnace is operated with small power input, it is desirable to decrease the current and still operate with as high a voltage as possible.

2. Inductance Factor

By definition, χ is the ratio of the inductive resistance, expressed in ohms, to the ohmic resistance losses. Obviously (Fig. 131), this is the tangent of the phase angle in open circuit. The arc is always considered as a purely ohmic resistance and therefore the power factor of the furnace under load depends directly on χ.

The higher χ becomes, the higher will $\eta_{W_{U_{max}}}$ be and the higher will be the efficiency at which the maximum useful input is obtained. This is likely to be misleading, since it appears to make a high inductive loss desirable from the power viewpoint. However, with increasing χ, the input drops more rapidly than efficiency increases, resulting in a drop in the maximum useful load with increasing χ. A large χ is desirable only in connection with cutting down the peak current.

χ can become small either if r_{Li} becomes small or if r_{LR} becomes large. Of course a small χ is desirable only if accomplished by a small r_{Li}: a large r_{LR} would offset the advantage, as is obvious from the formulas for W and W_U (where the values are given as product of Wr_{LR}/E^2). W and W_U are inversely proportional to r_{LR}.

For a given mechanical design of transformer, busses, and electrodes, χ is proportional to the frequency. r_{Li} was defined as $r_{Li} = 2\,\pi fL$ (page 170), where L is the inductance (in henrys). The smaller f is, the smaller are r_{Li} and χ. The influence of frequency will be discussed on page 181.

3. Voltage

In the general formulas for total power as well as for useful power (Eqs. 62a and 59a) the square of the voltage appears in the denominator. The expression, r_{LR}/E^2, has the dimension, power^{-1} (*e. g.*, 1/watt). Thus the expressions for W and W_U (right side of the equations) become dimensionless. For a given furnace design, the power and the useful power are proportional to the square of the voltage. This proportionality holds for identical values of ψ and χ. The maximum values $W_{U_{max}}$, W_{max}, $W_{U_{W_{max}}}$, and $W_{W_{U_{max}}}$ are functions only of χ, not of ψ, and therefore will increase with the square of the voltage so long as χ remains unchanged. This is most important. An increase of voltage leaves (after deduction of the voltage drop to cover the losses) a higher voltage for the arc. The arc resistance changes and therefore the value of ψ changes. The straight proportionality between power and square of voltage found for constant ψ does not apply to a given current.

The characteristic figures ($W_{W_{U_{max}}}$, W_{max}, $W_{U_{max}}$, and $W_{U_{W_{max}}}$) are independent of ψ and depend only on χ, which in turn is a function of furnace design. Consequently, these four quantities increase and decrease directly with the square of the applied voltage. For example, a furnace has a maximum power input of 6000 kw and a maximum useful power input of 5000 kw at 200 v. The current is 21,000 amp. If the voltage is raised to 250 v, W_{max} is raised $(250/200)^2 \times 6000 = 9390$ kw, $W_{U_{max}}$ to $(250/200)^2 \times 5000 = 7812.5$ kw, and the current to 26,250 amp. The same furnace may draw (at a different position of the electrodes) a current of 13,000 amp and a power (W) of 4000 kw at 200 v. Raising the voltage at 250 v would of course change again the current

and therewith also the resistance of the arc: ψ would have a different value and the new power could not be calculated directly from Equation (59a).

The load-current diagram is better suited for the study of individual furnaces than the diagrams based on χ and ψ. But the most important points in the diagram (W_{max}, $W_{U_{max}}$) are independent of ψ and are, as shown, directly proportional to the square of the voltage. Therefore the tendency to increase the operating voltage of arc furnaces is understandable. Increasing the arc voltage automatically increases the length of the arc. This in turn causes a heavy wear on the refractory (page 75), which, coupled with the danger to the operator, limits the possible increase of voltage. At present a potential of 275–300 v (sometimes 325 v) is considered to be the upper limit.

4. Frequency

The frequency influences all characteristic figures of the arc furnace. As mentioned above, the value of χ is proportional to it. For instance, with a furnace of $\chi = 3$ at 60 cycles, decreasing the frequency to 25 cycles will lower the value of χ to $25/60 \times 3 = 1.25$. For $\chi = 1.25$, $\eta_{W_{U_{max}}}$ is 0.615 and for $\chi = 3$, $\eta_{W_{U_{max}}}$ is 0.760 (from Equation 67a). Hence, the efficiency obtained at maximum useful power (*i. e.*, the normal operating efficiency) is decreased from 76 to 61.5%, or about 14%, by reducing the frequency from 60 to 25 cycles. (Had the original value of χ been 2 or 4 instead of 3, $\eta_{W_{U_{max}}}$ would have been 0.693 and 0.806, respectively. The change in frequency would change the values as follows: $\eta_{W_{U_{max}}} = 0.565$ and 0.666 for 25 cycles. The higher the original value of χ, the smaller is the relative influence of the frequency.)

TABLE XVIII

EXAMPLE OF INFLUENCE OF FREQUENCY AND INDUCTIVE RESISTANCE ON MAXIMUM LOAD AND MAXIMUM USEFUL LOAD

Load	$\chi = 2$ at 60 cycles			$\chi = 3$ at 60 cycles			$\chi = 4$ at 60 cycles		
	$f = 60$	$f = 25$	Increase	$f = 60$	$f = 25$	Increase	$f = 60$	$f = 25$	Increase
$r_{LR} \cdot \frac{W_{max}}{E^2}$	0.250	0.60	140%	0.167	0.400	140%	0.125	0.300	140%
$r_{LR} \cdot \frac{W_{U_{max}}}{E^2}$	0.1545	0.2175	41%	0.1201	0.1921	59%	0.0976	0.170	74%

The maximum useful load, however, increases with smaller χ (smaller frequency); the same is true for the total load. This is illustrated in Table XVIII.

E. Influence of the Transformer

So far, the voltage and current used in the calculations refer to the secondary of the furnace transformer. The transformer itself also has ohmic losses and an inductive voltage drop, both in the primary and in the secondary winding. Moreover, the magnetic circuit (core) causes additional ohmic losses and a phase shift, due to the magnetizing current. In fact, the power factors measured on the primary and the secondary side of the transformer are not the same.

Voltage drops and losses caused both by the primary and the secondary coils of the transformer can be readily incorporated into the calculation presented above. However, the core losses and the magnetization losses do not fit exactly into the picture. It is common practice to represent a transformer by an equivalent circuit, in which all resistances and inductances in each part of the transformer are considered lumped into equivalent resistors and reactors. Such an equivalent circuit is shown in Figure 132. The load can be of any nature whatsoever. In an arc furnace, the load consists of resistances and inductances in series (Fig. 132B). Obviously all resistances and inductances except e and f can be combined. It is necessary to refer all values of the transformer to the primary or secondary, by multiplying or dividing the values of the other side with the square of the ratio of windings.

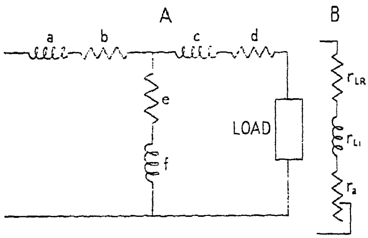


Fig. 132.—Circuit diagram for furnace with transformer: A, total diagram; B, load. Primary winding: a, inductance; b, resistance. Secondary winding: c, inductance; d, resistance. Magnetic circuit: e, core losses; f, inductance for magnetizing current.

For example, consider a transformer and furnace having the following values (a to d refer to Fig. 132): $a = 0.4$ ohm, $b = 0.0790$ ohm, $c = 10^{-4}$ ohm, $d = 2.21 \times 10^{-5}$ ohm, $r_{LR} = 5.10^{-5}$ ohm, $r_{Li} = 2.10^{-3}$ ohm, and $r_a = 0.0021$ ohm. In order to be able to make consistent calculations, all values have to be reduced either to the primary side or to the secondary. Assuming the transformer has a primary voltage of 11000 v and a secondary of 200 v, then the ratio of windings is 11000/200 = 55. The square of the ratio (55^2) is 3025. If all values are to be referred to the *primary* side, then the values for the secondary must be *multiplied* by 3025: $c' = 3025 \times 10^{-4} = 0.3025$, $d' = 3025 \times 2.21 \times 10^{-5} = 0.0677$, $r_{LR} = 3025 \times 5 \times 10^{-5} = 0.15125$, $r_{Li} = 3025 \times 2 \times 10^{-3} = 6.05$, and $r_a = 3025 \times 2.1 \times 10^{-3} = 6.35$. If all values are to be referred to the *secondary* side, then the values for the primary must be *divided* by 3025: $a' = 1/3025 \times 0.4 = 1.32 \times 10^{-4}$, and $b' = 1/3025 \times 0.079 = 2.61 \times 10^{-5}$.

Introduction of the core losses (e, f, Fig. 132) would greatly complicate matters. Of course the furnace may be considered as a load on the transformer. The circular diagram of the transformer can be readily applied and permits the introduction of the core losses, if so desired. For a general explanation of the circular diagram see Kapp.[76] The application of the circular diagram to the arc furnace has been explained by Wotschke.[77]

F. Three-Phase Furnaces

1. Symmetric

The deductions and equations of the previous chapter apply directly only to single-phase furnaces. However, the majority of the furnaces are connected to three-phase systems. Assuming a symmetric supply system and furnace design and a symmetric arrangement of busses, the charts and equations call for small changes only. If all designations of resistances and inductances (r_a, r_{LR}, r_{Li}) refer to the values of one phase, and if, moreover, E designates the line voltage, then the phase current, I, becomes (compare with Eq. 58):

$$I = \frac{E}{\sqrt{3}\sqrt{r_{Li}^2 + (r_{LR} + r_a)^2}} \tag{58b}$$

The total useful power is (compare with page 171):

$$W_U = 3\,I^2 r_a = \frac{E^2 r_a}{r_{Li}^2 + (r_{LR} + r_a)^2} \tag{59b}$$

Similarly, the total power is (compare with page 171):

$$W = 3\,I^2(r_{LR} + r_a) = \frac{E^2(r_{LR} + r_a)}{r_{Li}^2 + (r_{LR} + r_a)^2} \tag{62b}$$

It is evident that, with the chosen definition (E as line voltage and all resistance values referring to one phase), all equations except that for I are the same for single-phase and for three-phase furnaces. In general terms, the equation for I reads (compare with page 175):

$$\frac{I\sqrt{_{LR}}}{E\cancel{r_{LR}}} = \frac{1}{\sqrt{3}}\sqrt{\frac{1}{(1 + \psi)^2 + \chi^2}} \tag{58c}$$

Therefore the charts developed for the single-phase furnace can also be used for the symmetric three-phase furnace with the exception of the chart for the current (Fig. 129). A second scale applying to three-phase current has therefore been added in this figure, the three-phase current being $1:\sqrt{3}$ smaller than the single-phase current.

[76] G. Kapp, *Transformers*. Pitman, New York, 1925, p. 177.

[77] J. Wotschke, *Grundlagen des elektrischen Schmelzofens*. Knapp, Halle, 1933, p. 69.

2. Asymmetric

In certain instances, supply systems feeding into electric furnaces are not symmetric. But even with perfectly symmetric systems and transformers the loads of furnaces can be asymmetric. The busses can be arranged asymmetrically; the electrodes can burn off unevenly; or, finally, the control of the three phases can work unevenly. In view of the large influence of the inductance and resistance of the whole system, uneven power absorption and electrode consumption result. Figure 133

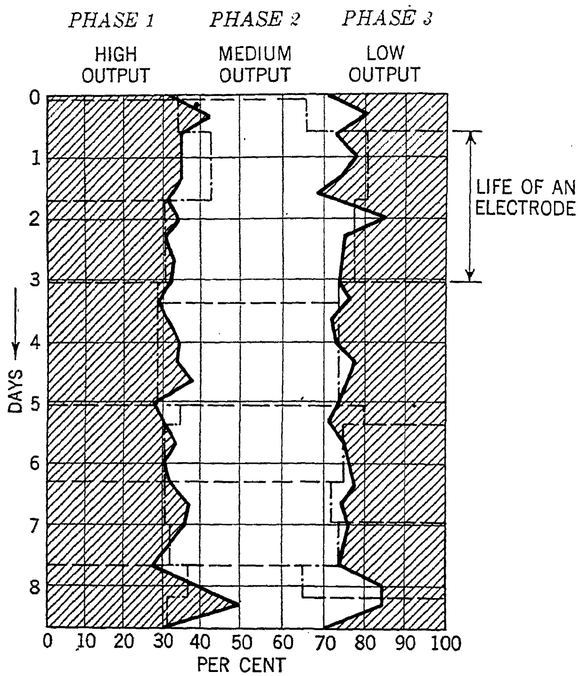


FIG. 133.—Output and electrode consumption of symmetric three-phase arc furnace.[78]

may serve as an example. The total power consumption and the electrode consumption are called 100% each and plotted *vs.* time—9 consecutive working days. The lines show a breakdown of power—and electrode consumption for the three phases. It is clear that the (left) phase 1 absorbs power, which is taken away from phase 3 (right). The figures for phase 2 can be considered as mean values.

Of course such asymmetry hinders full use of the available material. Special care is therefore taken to arrange the busses and electrodes as

[78] J. Wotschke, *Grundlagen des elektrischen Schmelzofens.* Knapp, Halle, 1933, p. 148.

symmetrically as possible. Since uneven distribution can, however, not always be avoided, every attempt is made to obtain uniform load, notwithstanding nonuniform resistances. Two ways are open to achieve this end: uniform voltage with nonuniform current; and uniform currents obtained by applying different voltages in each phase. The formula from which to calculate currents (for the first case) or voltages (for the second) are very involved. The first method is discussed in detail by Andreae,[79] the latter, by Wotschke.[78]

G. Current Diagram

The value of ψ changes with the length of the arc and with the voltage. Therefore the general diagrams (Figs. 126 to 129) do not lend themselves readily to the study of the behavior of individual furnaces. For such studies, diagrams with current as abscissa are better suited. In a given furnace (with a given voltage and frequency), the current is limited by r_{Li} and r_{LR}. The current can never become greater than the short-circuit current, which can be found from Equation (58) by setting $r_a = 0$ (compare with page 175):

$$I_{max} = \frac{E}{\sqrt{r_{LR}^2 + r_{Li}^2}} \tag{76a}$$

The minimum current is of course zero (if the arc resistance is infinity). Thus the length of the abscissa of the diagram is given. Against each value of I, the values of W, W_U, η and $\cos\varphi$ are plotted. In order to calculate the individual values from Equations (59), (62), (66), and (68), the value of r_a should be known. It can be found as function of I from Equation (58):

$$r_a = \sqrt{\frac{E^2}{I^2} - r_{Li}^2} - r_{LR} \tag{58d}$$

Introducing this value in the above equations, the variables are as follows:

$$W = I\sqrt{E^2 - I^2 r_{Li}^2} \tag{62c}$$

$$W_U = I\sqrt{(E^2 - I^2 r_{Li}^2)} - I r_{LR} \tag{59c}$$

$$\eta = 1 - \frac{I r_{LR}}{\sqrt{E^2 - I^2 r_{Li}^2}} \tag{66c}$$

$$\cos\varphi = \sqrt{1 - \frac{I^2 r_{Li}^2}{E^2}} \tag{68c}$$

The maximum values are, as stated, independent of the load. They are functions of χ only.

[79] F. V. Andreae, *Trans. Am. Electrochem. Soc.*, 42, 50 (1923).

For the symmetric three-phase furnace, the equation for current has been given on page 183. From this equation, r_a can again be computed as function of I:

$$r_a = \sqrt{\frac{E^2}{3\,I^2} - r_{Li}^2} - r_{LR} \tag{58e}$$

The useful power is:

$$W_u = I\sqrt{3}\,(\sqrt{E^2 - (I\sqrt{3}\,r_{Li})^2} - I\sqrt{3}\,r_{LR}) \tag{59d}$$

The total power is:

$$W = I\sqrt{3}\,\sqrt{E^2 - (I\sqrt{3}\,r_{Li})^2} \tag{62d}$$

The efficiency is:

$$\eta = \frac{W_u}{W} = \frac{\sqrt{E^2 - (I\sqrt{3}\,r_{Li})^2} - I\sqrt{3}\,r_{LR}}{\sqrt{E^2 - (I\sqrt{3}\,r_{Li})^2}}$$

$$= 1 - \frac{1}{\sqrt{\left(\frac{E}{I\sqrt{3}\,r_{LR}}\right)^2 - \left(\frac{r_{Li}}{r_{LR}}\right)^2}} \tag{66d}$$

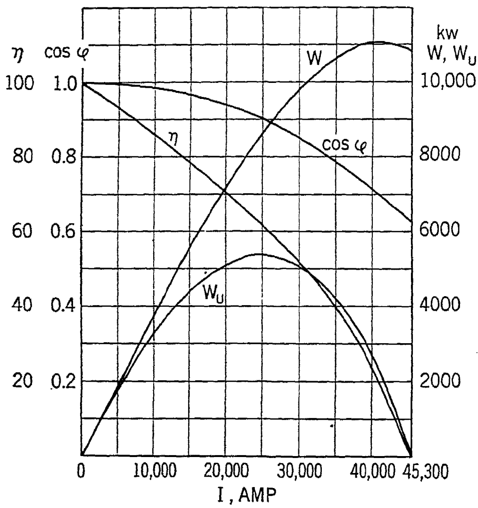


FIG. 134.—Characteristic curves of arc furnaces. 220 v, $r_{Li} = 2.19 \times 10^{-3}$ ohm, 60 cycles, $r_{LR} = 1.75 \times 10^{-3}$ ohm, $\chi = 1.255$.

The total apparent power is:

$$W_a = EI\sqrt{3}$$

The power factor may be defined as:

$$\cos\varphi = \frac{W}{W_a} = \sqrt{1 - \left(\frac{I\sqrt{3}\,r_{Li}}{E}\right)^2} \tag{68d}$$

In Table XIX, the formulas for the different important variables are summarized. Some formulas have been added which are not specifically mentioned in the text, in order to make the tabulation more complete.

In order better to understand the arc furnace diagram based on current a practical example is given:

For a three-phase symmetric furnace, line voltage 220 v, 60 cycles, inductance per phase 5.8×10^{-6} henry, $r_{Li} = 2\pi \times 60 \times 5.8 \times 10^{-6} = 2.19 \times 10^{-3}$ ohm. The resistance per phase, r_{LR}, is 1.75×10^{-3} ohm, and the following values can be calculated:

$$\chi = \frac{2.19 \times 10^{-3}}{1.75 \times 10^{-3}} = 1.255$$

$$I_{max} = \frac{220}{\sqrt{3}\sqrt{(2.19 \times 10^{-3})^2 + (1.75 \times 10^{-3})^2}} = \frac{220 \times \cancel{10^{-3}}\, 10^{3}}{4.87} = 45{,}300 \text{ amp}$$

$$W_{U_{max}} = \frac{E^2}{r_{LR}} \cdot \frac{1}{2(\sqrt{\chi^2 + 1} + 1)} = \frac{48{,}400}{1.75 \times 10^{-3}} \cdot \frac{1}{2\sqrt{2.57} + 2} \cancel{\times 10^{3}}$$

$$= 5320 \text{ kw and occurs at } 25{,}200 \text{ amp}$$

$$W_{max} = \frac{E^2 \times \cancel{10^{3}}}{2\chi r_{LR}} = \frac{48{,}400}{1.75 \times 10^{-3} \times 2.51} = 11{,}000 \text{ kw and occurs at } 41{,}200 \text{ amp}$$

$$\eta_{W_{U_{max}}} = \frac{\chi^2 + 1}{1 + \sqrt{\chi^2 + 1}} = \frac{1.255^2 + 1}{1 + \sqrt{1 + 1.255^2}} = 0.616$$

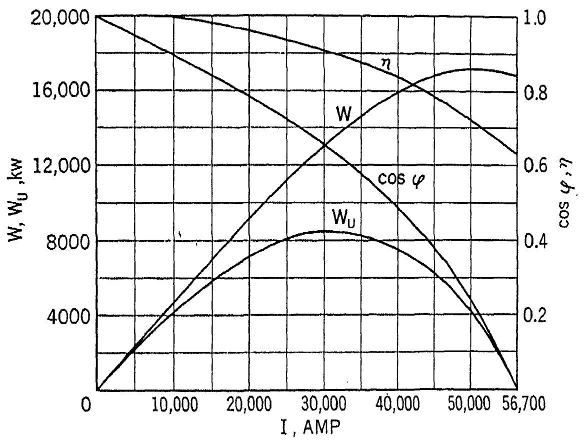


FIG. 135.—Characteristic curves of arc furnaces. Same furnace as for Figure 134, but operated at 275 v.

The current diagram can now be drawn (Fig. 134). The current is plotted along the abscissa. Two vertical scales are used, one for the total and the useful power in kw, the other—dimensionless—for η and $\cos \varphi$. In the diagram, the maximum of the power can be readily seen.

TABLE XIX

REVIEW OF EQUATIONS ON ARC FURNACES

Item (Magnitude)	Noninductive furnace: Straight formula	Eq. No. / Page	Reduced formula	Eq. No. / Page	Single-phase furnace: Straight formula	Eq. No. / Page	Reduced formula	Eq. No. / Page	Formula for current diagram	Eq. No. / Page
W (Total Power)	$\frac{E^2}{r_a+r_L}$	$\frac{52}{167}$	$\frac{1}{\psi+1}$	$\frac{52a}{169}$	$\frac{E^2(r_{LR}+r_a)}{r^2_{Li}+(r_{LR}+r_a)^2}$	$\frac{62}{171}$	$\frac{\psi+1}{\chi^2+(1+\psi)^2}$	$\frac{62a}{172}$	$I\sqrt{E^2-I^2r^2_{Li}}$	$\frac{62c}{185}$
W_U (Useful Power)	$\frac{r_aE^2}{(r_a+r_L)^2}$	$\frac{51a}{167}$	$\frac{\psi}{(\psi+1)^2}$	$\frac{53a}{169}$	$\frac{E^2r_a}{r^2_{Li}+(r_{LR}+r_a)^2}$	$\frac{59}{171}$	$\frac{\psi}{\chi^2+(1+\psi)^2}$	$\frac{59a}{172}$	$I\sqrt{E^2-I^2r^2_{Li}}-Ir_{LR}$	$\frac{59c}{185}$
$W_{\max}$	$\frac{E^2}{r_L}$	$\frac{53}{168}$	1		$\frac{E^2}{2r_{Li}}$	$\frac{64}{171}$	$\frac{1}{2\chi}$	$\frac{64a}{175}$		
Condition for $W=W_{\max}$	$r_a=0$	$\frac{\ }{168}$			$r_{Li}=r_{LR}+r_a$	$\frac{63}{171}$	$\chi=\psi+1$	$\frac{63a}{174}$		
$W_{U_{\max}}$	$\frac{E^2}{4r_L}$	$\frac{55}{168}$	0.25		$\frac{E^2\sqrt{r^2_{Li}+r^2_{LR}}}{r^2_{Li}+(\sqrt{r^2_{Li}+r^2_{LR}}+r_{LR})^2}$	$\frac{61}{170}$	$\frac{1}{2(\sqrt{\chi^2+1}+1)}$	$\frac{61a}{174}$		
Condition for $W_U=W_{U_{\max}}$	$r_a=r_L$	$\frac{\ }{167}$	$\psi=1$		$r_a=\sqrt{r^2_{Li}+r^2_{LR}}$	$\frac{60}{171}$	$\psi=\sqrt{\chi^2+1}$	$\frac{60a}{174}$		
$W_{W_{U_{\max}}}$	$\frac{E^2}{2r_L}$	$\frac{54}{168}$	0.50		$\frac{E^2(r_{LR}+\sqrt{r^2_{Li}+r^2_{LR}})}{r^2_{Li}+(\sqrt{r^2_{Li}+r^2_{LR}}+r_{LR})^2}$	$\frac{65}{172}$	$\frac{\sqrt{\chi^2+1}+1}{2[(\chi^2+1)+\sqrt{\chi^2+1}]}$	$\frac{65a}{174}$		
η (Efficiency)	$\frac{r_a}{r_a+r_L}$	$\frac{57}{169}$	$\frac{\psi}{\psi+1}$	$\frac{57a}{169}$	$\frac{r_a}{r_{LR}+r_a}$	$\frac{66}{172}$	$\frac{\psi}{\psi+1}$	$\frac{66a}{172}$	$1-\frac{Ir_{LR}}{\sqrt{E^2-I^2r^2_{Li}}}$	$\frac{66c}{185}$
$\eta_{W_{U_{\max}}}$ (Efficiency of Max. Useful Power)	0.50	$\frac{\ }{168}$			$\frac{\sqrt{r^2_{Li}+r^2_{LR}}}{r_{LR}+\sqrt{r^2_{Li}+r^2_{LR}}}$	$\frac{67}{172}$	$\frac{\sqrt{\chi^2+1}}{1+\sqrt{\chi^2+1}}$	$\frac{67a}{174}$		
I (Current)	$\frac{E}{r_L+r_a}$	$\frac{48}{167}$			$\frac{E}{\sqrt{r^2_{Li}+(r_{LR}+r_a)^2}}$	$\frac{58}{171}$	$\sqrt{\frac{1}{(1+\psi)^2+\chi^2}}$	$\frac{58a}{175}$	$r_a=\sqrt{\frac{E^2}{I^2}-r^2_{Li}}-r_{LR}$	$\frac{58d}{185}$
$\frac{I_{\max}}{I_{W_{U_{\max}}}}$							$\sqrt{2\left(1+\frac{1}{\sqrt{1+\chi^2}}\right)}$	$\frac{77}{176}$		
Cos φ (Power Factor)					$\frac{r_{LR}+r_a}{\sqrt{r_{Li}+(r_{LR}+r_a)^2}}$	$\frac{68}{172}$	$\frac{\psi+1}{\sqrt{\chi^2+(\psi+1)^2}}$	$\frac{68}{172}$	$\sqrt{1-\frac{I^2r^2_{Li}}{E^2}}$	$\frac{68c}{185}$

TABLE XIX—CONTINUED

Item (magnitude)	Three-phase furnace					
	Straight formula	Eq. No. / Page	Reduced formula	Eq. No. / Page	Formula for current diagram	Eq. No. / Page
W (Total Power)	$\frac{E^2(r_{LR}+r_a)}{r^2_{Li}+(r_{LR}+r_a)^2}$	62 / 171	$\frac{\psi+1}{\chi^2+(1+\psi)^2}$	62a / 172	$I\sqrt{3}\sqrt{E^2-(I\sqrt{3}r_{Li})^2}$	62d / 186
W_U (Useful Power)	$\frac{E^2 r_a}{r^2_{Li}+(r_{LR}+r_a)^2}$	59 / 171	$\frac{\psi}{\chi^2+(1+\psi)^2}$	59a / 172	$I\sqrt{3}(\sqrt{E^2-(I\sqrt{3}r_{Li})^2}-r_{LR})$	59d / 186
W_{max}	$\frac{E^2}{2r_{Li}}$	64 / 171	$\frac{1}{2\chi}$	64a / 175	$\sqrt{1-\left(\frac{I\sqrt{3}r_{Li}}{E}\right)^2}$	68d / 186
Condition for $W=W_{max}$	$r_{Li}=r_{LR}+r_a$	63 / 171	$\chi=\psi+1$	63a / 174		
$W_{U_{max}}$	$\frac{E^2\sqrt{r^2_{Li}+r^2_{LR}}}{r^2_{Li}+(\sqrt{r^2_{Li}+r^2_{LR}}+r_{LR})^2}$	61 / 171	$\frac{1}{2(\sqrt{\chi^2+1}+1)}$	61a / 174		
Condition for $W_U=W_{U_{max}}$	$r_a=\sqrt{r^2_{Li}+r^2_{LR}}$	60 / 171	$\psi=\sqrt{\chi^2+1}$	60a / 174		
$W_{W_{U_{max}}}$	$\frac{E^2(r_{LR}+\sqrt{r^2_{Li}+r^2_{LR}})}{r^2_{Li}+(\sqrt{r^2_{Li}+r^2_{LR}}+r_{LR})^2}$	65 / 172	$\frac{\sqrt{\chi^2+1}+1}{2[(\chi^2+1)+\sqrt{\chi^2+1}]}$	65a / 174		
η (Efficiency)	$\frac{r_a}{r_{LR}+r_a}$	66 / 172	$\frac{\psi}{\psi+1}$	66a / 172	$1-\frac{1}{\sqrt{\left(\frac{E}{I\sqrt{3}r_{LR}}\right)^2-\left(\frac{r_{Li}}{r_{LR}}\right)^2}}$	66d / 186
$\eta_{W_{U_{max}}}$ (Efficiency of Max. Useful Power)	$\frac{\sqrt{r^2_{Li}+r^2_{LR}}}{r_{LR}+\sqrt{r^2_{Li}+r^2_{LR}}}$	67 / 172	$\frac{\sqrt{\chi^2+1}}{1+\sqrt{\chi^2+1}}$	67a / 174		
I (Current)	$\frac{E}{\sqrt{3}\sqrt{r^2_{Li}+(r_{LR}+r_a)^2}}$	58b / 183	$\frac{1}{\sqrt{3}}\sqrt{\frac{1}{(1+\psi)^2+\chi^2}}$	58c / 183	$r_a=\sqrt{\frac{E^2}{3I^2}-r^2_{Li}}-r_{LR}$	58e / 186
$\frac{I_{max}}{I_{W_{U_{max}}}}$			$\sqrt{2\left(1+\frac{1}{\sqrt{1+\chi^2}}\right)}$	77 / 176		
$\cos\varphi$ (Power Factor)	$\frac{r_{LR}+r_a}{\sqrt{r^2_{Li}+(r_{LR}+r_a)^2}}$	68 / 172	$\frac{\psi+1}{\sqrt{\chi^2+(\psi+1)^2}}$	68a / 194		

The line voltage was a very important factor in determining the curves in Figure 134. A first brief glance at Equations (59a) and (62c) indicates an increase of power with the square of the voltage. However, r_a is not independent of the voltage and therefore no straight proportionality occurs.

For comparison, two other graphs are added, Figures 135 and 136, for 275 v and 110 v, respectively. The maximum useful power and the maximum power are directly proportional to the square of the voltage; but with different voltages they occur with different currents. A comparison of Figure 134 (220 v) with Figure 135 (275 v) and Figure 136 (110 v) shows that, not only is the maximum power higher with increased

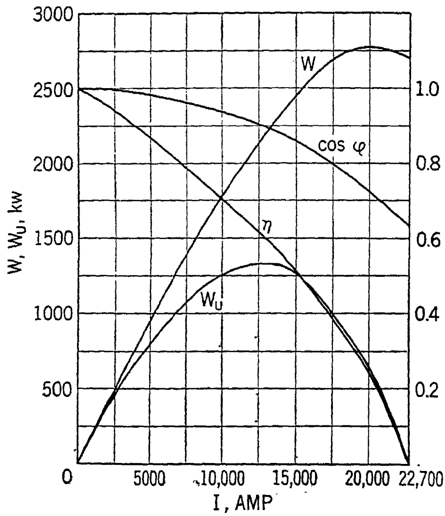


FIG. 136.—Characteristic curves of arc furnaces. Same furnace as for Figure 134, but operated at 110 v.

voltage, but for a given constant power the efficiency is higher if produced with higher voltage. Therefore, as a general rule, it may be stated that arc furnaces should be built with as high a voltage as possible. But high voltages mean long arcs and this effect is one limitation to increasing the voltage: long arcs wear out the lining quickly.

Previously, the influence of χ was examined (page 179) and it was stated that χ is directly proportional to the frequency on which the furnace operates. The influence of the frequency will be still more clear from Figure 137, which shows the characteristic curves (power, efficiency, power factor) of the same furnace analyzed in Figure 134, under the assumption that it is operated at 25 cycles. It is worth noting that the

maximum power, the useful power, and the efficiency are much higher with lower frequencies. To operate large furnaces at reduced frequencies is worth considering.

H. Voltage Surge. Light Flickers

The resistance of the arc changes within the limits 0 and ∞. Consequently the current changes from 0 to I_{max}. These heavy fluctuations of current appear (though somewhat reduced by the reactance of the magnetizing losses) on the primary of the transformer. Some changes occur slowly and possibly over a period of hours, others very rapidly in fractions of a second (*e. g.*, if a pile of scraps collapses in the furnace).

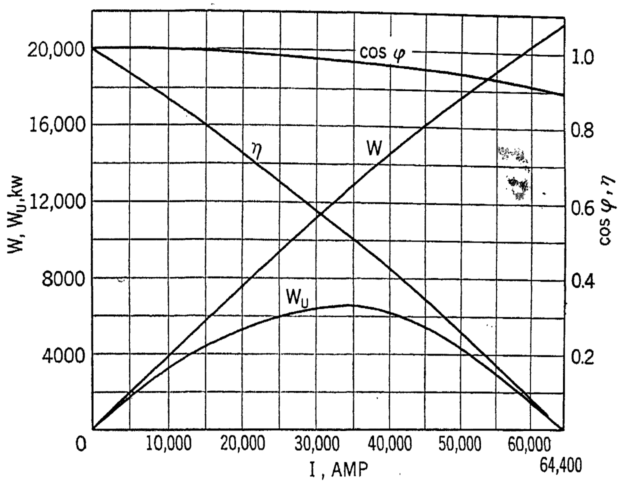


FIG. 137.—Characteristic curves of arc furnaces. Same furnace as for Figure 134, but operated at 220 v, 25 cycles.

The ratio, $I_{max}/I_{W_{U_{max}}}$ (Eq. 77, page 176), which characterizes the increase of current, gives only part of the entire picture. The furnace of course does not always operate on the current "$I_{W_{U_{max}}}$" and therefore the peaks can become (in proportion to the momentary value of current) very much higher than that expressed by Equation (77). The fact that the furnaces are frequently not operated at $W_{U_{max}}$ is often overlooked and therefore, in the literature, the required limit of the current peak is given as 2.5–3. It would be desirable to operate the furnaces as far as possible at $W_{U_{max}}$, thus limiting the possible peak to that given by Equation (77). Moreover, the reactor usually has several taps and therefore the value of x may change at different times of operation. Then Equation (77), which was found by dividing the value of Equation (76) by that of (74), be-

comes meaningless. The equations are repeated here for convenience:

$$\frac{I_{max}}{I_{WU_{max}}} = \sqrt{2\left(1 + \frac{1}{\sqrt{1 + \chi^2}}\right)} \tag{77}$$

$$\frac{I_{max} r_{LR}}{E} = \sqrt{\frac{1}{1 + \chi^2}} \tag{76}$$

$$\frac{I_{WU_{max}} r_{LR}}{E} = \sqrt{\frac{1}{2[(1 + \chi^2) + \sqrt{1 + \chi^2}]}} \tag{74}$$

However the highest absolute peaks of current are reached during the melting down period, in which the normal operating current is more nearly $I_{WU_{max}}$ than it is during the refining period. This of course tends to bring down the relative value of the peak current.

These more or less rapid changes in current are one of the two important causes of voltage surges. The other source of surges is to be seen in the breaking of the current by the main circuit breaker. The tap-changing switch, which is used for setting the transformer voltage and the reactor, usually should not be operated under load. Before selecting a new position, therefore, the main circuit breaker is operated. Whitehouse and Levy (*loc. cit.*, page 159) have found that the circuit breaker in arc furnaces is interrupted from 30 to 100 times a day. If the switching off coincides with extinguishing of the arc, surges appear. Most surges due to switching do not occur in all three phases: in fact 72% occur in one phase only, 25% in two phases, and only 2% in all three. Failure of the circuit breaker (because of surges on the load side) have been reported.[80]

The changes in current either from full to zero or from one value to the short-circuit value (I_{max}) result in voltage surges. They are dangerous from two viewpoints: they may endanger the transformer windings as any short circuit would; or they can inconvenience and endanger the supply system. The influence of the voltage surges on the transformer itself is met by designing sufficiently strong coils and reinforcements.

Their influence on the supply lines and the means of combating the formation of surges have been investigated repeatedly. The surges cause double danger in the lines: by breaking down insulation; and, if they occur frequently, even in limited amount, by causing flicker in the line voltage. This flicker is extremely inconvenient for the users of light.

Generally speaking, short surges lasting only fractions of a cycle are dangerous to the equipment, while surges lasting three cycles or more are disturbing because of flicker. Flicker of light is caused by changes of voltage, the changes becoming objectionable when the flicker becomes

[80] L. V. Black and E. W. Boehne, *Iron Steel Engr.*, 18, 88 (1941).

perceptible. The amount of voltage fluctuation perceptible in electric lamps depends on a number of factors: the standard voltage, around which fluctuations occur; duration of the individual fluctuation; type of lamp involved; frequency of occurrence of fluctuations; and the human element (physiological character, fatigue of the observer, etc.). The limit of perceptibility is often plotted against the frequency of occurrence of fluctuations, the chart holding for one type of lamp and one standard voltage. In order to eliminate the human element, the results of a large number of experiments are combined by statistical methods. It is remarkable how well the various investigations (Starr and Falls, Fig. 2—Fig. 138 below—and Fig. 3;[81] Simons;[82] Schwabe [83]) check. They show

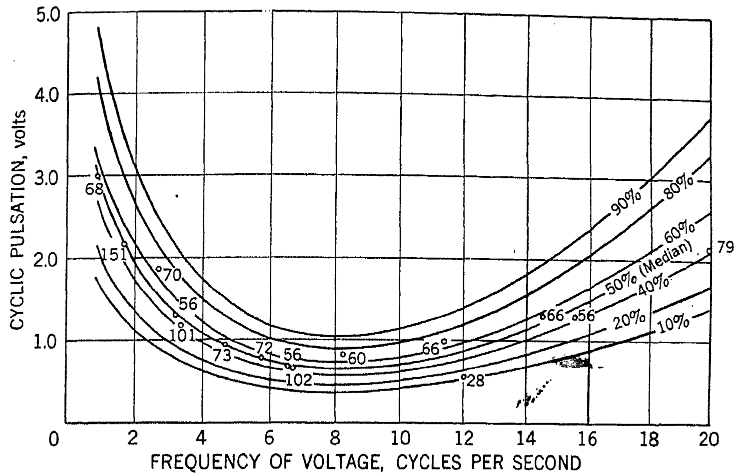


FIG. 138.—Sensitivity of the human eye to light flicker with condensers.[81]

that a frequency of pulsation of approximately 7–8 per second is a critical value. At this frequency, voltage fluctuations of as little as 0.5% are already perceptible. At lower or higher values of frequency, the sensitivity of the human eye seems to decrease. Figure 138, from Starr and Falls, has a set of such curves; the curves also include the "human element."

Voltage surges and changes causing flicker must be reduced. Surges can be reduced by capacitors or lightning arresters between line and ground. These can be put on the primary or the secondary side of the transformer. Another means of counteracting the surges is by increased reactance of the circuit. All investigators seem to agree that condensers

[81] F. M. Starr and O. B. Falls, *Iron Steel Engr.*, **18**, 30 (1941).
[82] K. Simons, *Elektrotech. Z.*, **38**, 453 (1917).
[83] E. Schwabe, *Elektrowärme*, **9**, 127 (1939).

are valuable in cutting down the severity and the number of surges;[72, 80, 84] but the usefulness of lightning arresters for this purpose is controversial.

Figure 139 shows a typical wiring diagram of a circuit in which condensers are used to counteract surges. By way of example, some of the results of Stewart and coworkers [84] can be given. They investigated

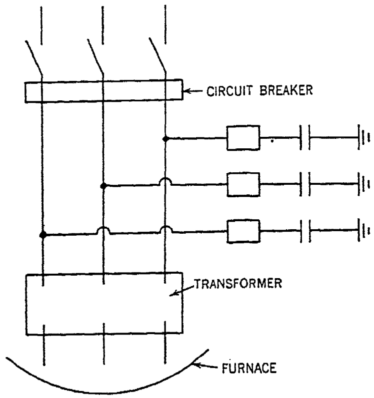


Fig. 139.—Wiring diagram for smoothing voltage surges.

various arrangements of lightning arresters and capacitors on the high- and low-voltage side of the furnace transformer. Addition of a reactor was also tried. Some of the more important findings follow:

(1) A determination of surges must be carried out over a considerable length of time because the occurrence of a maximum crest is unpredictable; it may come on the first day of a test or after weeks of operation. The crest of the highest surge was 58 kv line to ground, while the average of the six highest surges was only 31.1 kv.

(2) Indication of the maximum crest is unsatisfactory because of the statistical frequency of crests: in one specific case (low-voltage capacitors, lightning arresters, 5.9% reactor) the test was carried over 165 days; the daily average number of surges above 20 kv line-to-ground was 13. Thus the total number of observed surges was 2145.

(3) High-voltage surges of at least six times normal in the primary and possibly ten or more times normal in the secondary may occur in the operation of an electric furnace.

(4) Capacitors connected from line-to-ground on the high-voltage side of the furnace transformer reduce both number and magnitude of surge voltages. They also reduce the slope of the surge wave front. The sloping effect is of value for most insulations, particularly that of transformers.

The surges were determined by a clydonograph, which records surges lasting only a fraction of a cycle. All these protections tend to cut down the surges passed on to the supply system.

In any system, the amount of flicker or surges depends upon the location in the system. The impedance of the system helps to cut down

[84] W. R. Stewart, S. B. Griscom, and J. E. Hobson, *Westinghouse Engr.*, 1, 11 (1941).

the voltage changes coming from the furnace. Points close to the furnace get heavier changes than points further away which are protected by line impedance. In order to eliminate voltage flicker, series capacitors have in some instances proved successful. However, an analysis is necessary for each individual case. Jones and Stearns [85] show that, if two furnaces are put on the same supply, their flicker and surges do not add, even if both furnaces are melting at the same time. The flickers almost never coincide.

I. Conclusions

From the foregoing considerations a number of important conclusions can be drawn:

1. Method of Calculating Furnaces

Starting from the described output (ton per hour), the necessary holding time (metallurgical considerations), and the actual melting time, the size of the furnace can be determined: with a representing output (ton per hour), b holding time (hour), t melting time (hour per ton), and V holding capacity (ton), then $a = V/(t + b)$. In order to achieve the melting in a desired time (t), a certain connected load is necessary (Fig. 48, page 76). Because the power transmitted by one group of electrodes is limited, the amount of this connected load decides whether or not the production can be attained in one furnace.

After the connected load for the furnace has been selected, the voltage must be chosen. It has been shown that high voltages are desirable from the viewpoint of efficiency. Because of the safety for the operator and the increase of arc length with higher voltages, 325 v has been considered, so far, as the upper limit of voltage. No systematic study is available showing the connection between transformer size and furnace volume, as actually built. At present, arc furnaces are in general not built for maximum voltages lower than 200 v.

The operating current follows from the selected voltage. The furnace should normally be operated at $W_{U_{max}}$. If $W_{U_{max}}$ and E are known, the corresponding current can be found. The maximum current must next be determined, either directly or by selecting $I_{max}/I_{W_{U_{max}}}$. The maximum current in turn determines the inductive resistance. In large furnaces it may be hard to reach a sufficiently low value of inductance; in small furnaces inductance must be added separately. (In large furnaces, a separate reactor may be provided for the periods of working with reduced power.)

[85] B. M. Jones and C. M. Stearns, *Trans. Am. Inst. Elec. Engrs.*, **60**, 763 (1941).

2. Reactance in Steps

As the melting proceeds, the danger of sudden current peaks decreases, and the reactance can therefore be cut down. For this reason, it is desirable to keep the reactance of the transformer low, adding necessary reactance by means of a separate reactor.

It would be desirable to establish, for each value of the reactance and each available voltage, curves showing the useful power, the (electric) efficiency and the power factor as functions of the current. Such diagrams could be established only after the furnace has actually been installed because of the difficulty of calculating in advance the resistance and the reactance of the furnace circuit. The curves would have to be found by tests; but it would be highly desirable to determine them because they give the operator the only safe basis for operation.

3. Operation

For a given furnace there is, for each voltage and reactor value, one value of current which yields the highest amount of useful heat. This value cannot be found by observing the ammeter—high currents above a certain amount yield smaller power—nor even by observing the wattmeter. The point of $W_{U_{max}}$ (or more correctly, the points, because for every tap on transformer and reactor there is a relative $W_{U_{max}}$) must be determined separately.

The highest production can again only be obtained if $W_{U_{max}}$ is known. As melting proceeds, the power requirements become lower. Maintaining unchanged power would result only in burning off the refractory and electrodes at an unnecessarily fast rate. The lowering of the power can be brought about either by decreasing the voltage or by decreasing the current (with unchanged voltage). Decreasing the current (with unchanged voltage) is effected by lengthening the arc, which makes for lower power consumption (η, the electric efficiency, is "unity" at zero current) but causes shorter refractory life. No systematic investigations are known which give, for an actual case, the comparison of both factors. The change of efficiency by change of voltage and arc length can be calculated readily enough, but determination of the actual increase of power consumption considering dependent and proportional losses is not quite so simple (see page 200). Aside from an old German publication,[86] no compilation of roof and refractory life as a function of length of arc is known.

The operator of the furnace must base the adjustments of the control on some instrument. He can rely on the wattmeter or the ammeter, or (if available) the power-factor meter. The wattmeter is not a con-

[86] S. Kriz, *Stahl u. Eisen,* 49, 417 (1929).

venient measure of the behavior of the furnace because the same value can occur twice for each transformer voltage (once for a high and once for a low value of current). Either the ammeter or the power-factor meter is convenient. (The power factor does not show great changes over a considerable range of operations; for practical purposes, therefore, a power-factor meter is also not advisable in this connection; see page 187). The value to be maintained is, however, different for different taps of the reactor or the transformer, so that it is desirable to prepare the diagrams mentioned above based on current for the various steps, and to post at least the values of $W_{U_{max}}$ for all combinations of taps near the furnace to guide the operator. The diagrams can in almost all instances be set up only after carrying out certain measurements (see following section).

J. Measurements

Selection of the value of current or power factor at which to operate the furnace depends upon the furnace characteristics. In order to determine these from the formulas given in the text, the ohmic and inductive resistance must be measured.

Measurement of the total power (W), the reactance power and the apparent power under load, together with the current, permits the determination of $r_a + r_{LR}$ and of r_{Li}, but not the separation of r_a and of r_{LR}. See (*a*) below.

In some instances, it is possible to measure total power, reactance power, and apparent power, together with the current, with electrodes short-circuited—touching the bottom. Then it is possible to calculate r_{LR} and r_{Li}; in short-circuit, r_a is zero. See (*b*) below. However, it is desirable to find, not only the over-all values of ohmic and inductive resistances, but also to have the breakdown of these values, assigning to each part of the circuit (busses, solid part; busses, flexible cable; electrodes, etc.) a certain part of the total value of resistance. Such knowledge will permit change of the individual design in order to approach the desired values.

A breakdown of the ohmic resistance can be obtained by measuring it with direct current and applying the measurement to the individual parts of the circuit. See (*c*) below. A breakdown of the inductive resistance can be obtained by calculation, from a knowledge of the over-all inductive resistance, the voltage drop in the individual sections, and the ohmic resistance. Because of the high currents involved, measuring individual voltage drops in the various parts of the circuit calls for a special technique. See (*d*) below.

(*a*) The input on the primary (W) can be measured with either two or three wattmeters in the usual manner. The reactance power (N_{wl}) can be measured by a wattmeter method, connecting the voltage coil

appropriately. Figure 140 shows this connection.[87] From these measurements, $r_a + r_{LR}$ can be calculated (Eqs. 32, 32a, and 33, page 61); r_{Li} can be calculated by using the same formulas and introducing N_{wl} instead of W.

(*b*) In order to measure W and N_{wl} with shorted electrodes, the voltage must be reduced to such an extent that the current comes into the range of available instruments (ammeter and wattmeter) and will not damage reactor, transformer, or busses by overheating. In rare cases, the reduction of furnace voltage by use of the largest reactor steps

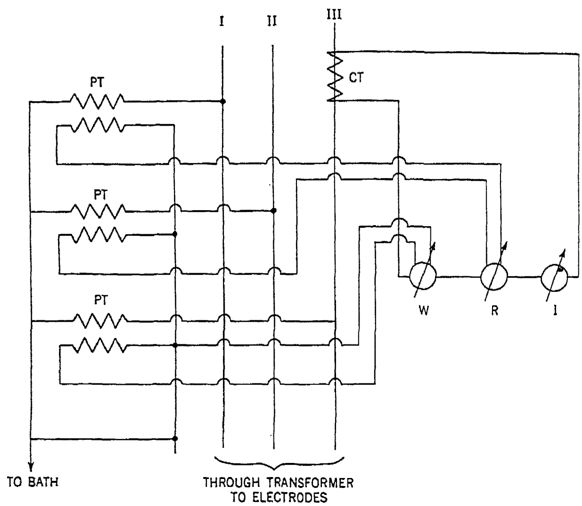


FIG. 140.—Wiring diagram for measuring circuit (measuring reactance power, after Schwabe and Fischer[87]). W, wattmeter; R, wattmeter for reactance power; I, ammeter; CT, current transformer; PT, potential transformer. The instruments are shown for phase III only. Identical circuits must be used for phases I and II.

is sufficient to make this kind of measurement possible. In most instances, however, this is not the case. And seldom only are there auxiliary transformers available to lower the voltage on the primary of the transformer sufficiently to make this method applicable. Then the following technique, consisting of determining separately the inductances in reactor, transformer, and furnace circuit, may be applied. From the known data of the reactor and transformer, the resistance and inductance of either can be calculated; and if the voltage in the furnace circuit cannot be lowered enough to make this measurement possible, it is necessary to measure W and N_{wl} on the secondary of the transformer at the high-voltage end of the busses. Such measurements are very difficult because

[87] E. Schwabe and W. Fischer, *Elektrowärme*, 9, 146 (1939).

to carry out endothermic reactions. The exothermic heat of reaction has been deducted. In other words, U_F designates the heat content of the load in the furnace at the moment of discharging. Wall losses will be considered later. First the flow of heat, ending as useful heat (U_F), must be followed.

In order to get U_F kwhr (or Btu) into the furnace chamber, more than U_F kwhr must enter the electrodes. The heat balance of the elec-

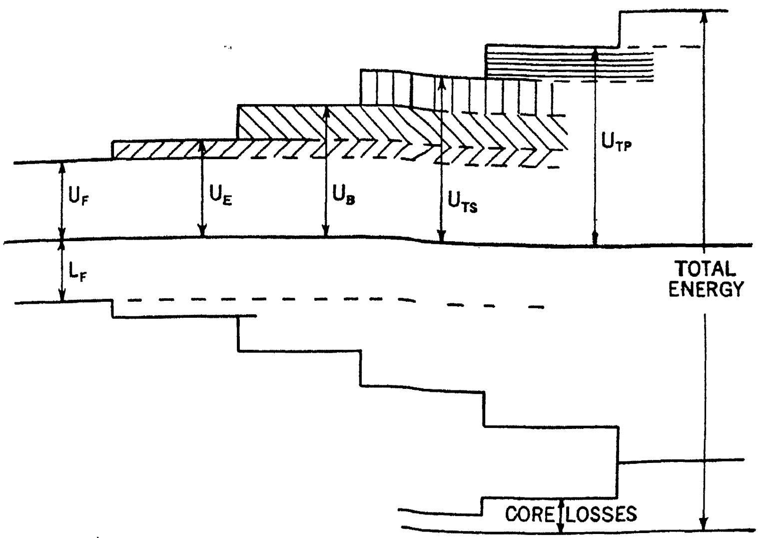


FIG. 144.—Energy balance considering independent, dependent, proportional and nonproportional losses.

trodes was discussed on page 130. At present, it is sufficient to remember that the actual energy losses through the electrodes occur at the water-cooled roof ring and above the roof. Instead of considering the entire electrode losses, only a part will be introduced here—that which is due to the "useful" current. Because of this part, U_E kwhr ($U_E > U_F$) must enter the electrodes if U_F is to leave them.

Following the flow of energy outward, the busses must be considered. Again the same rule applies: in order to pass U_E to the electrodes, U_B kwhr ($U_B > U_E$) must enter the busses. The difference, $U_B - U_E$, is an expression for the energy losses in the busses. These losses are proportional to the current passing through the busses and not to the voltage drop in the electrodes or to the potential. The various (electric) losses in the electrodes, busses, and transformer can be added algebraically. Now part of U_E is the useful load, U_F, and part the losses in the electrodes. Consequently, part of the losses ($U_B - U_E$) is due to

the useful load, U_F, and part to the electrode losses. In order to show the relationship, the part of $U_B - U_E$, which is due to the electrode losses, is shaded in the same way as $U_E - U_F$.

The energy flow next passes the secondary of the transformer. The same manner of indication is used to show the breakdown of losses in the secondary ($U_{TS} - U_B$) into those due to useful heat, to electrode losses and to losses in the busses. The transformer core causes losses independent of the load and not proportional to it. They therefore do not appear here. Still following the path of energy, the primary of the transformer (U_{TP}) and finally the reactor (U_R) must be considered.

At the start, only the useful energy, U_F, was introduced. However, the furnace body, being hot, causes considerable *heat* losses. These consist in part of the losses from the outside surface, and in part of water-cooling of the door, door support, etc. (Water-cooling of the electrodes should not be counted as heat loss, as will be explained later.) The losses of the furnace chamber, designated by L_F, are (in Fig. 144) drawn under the zero line. They are entirely independent of the useful heat. Following the flow of losses "upstream," so to speak—toward the transformer—the same picture is encountered as with the useful heat. In order to obtain the energy necessary to cover the wall losses, L_F, into the furnace, a larger stream of energy, L_R, must enter the reactor.

For the sake of completeness, a third, though small, stream must be added: the transformer core causes losses which are entirely independent of the load (or the wall losses). The core losses cause additional energy losses in the primary winding of the transformer and in the reactor.

In this presentation the losses in the individual elements (electrodes, busses, primary and secondary of the transformer, and reactor) are not shown together. The part of the losses necessary to carry the useful heat is, for each loss item, separated from the part necessary to cover the losses due to the wall losses. The wall losses are independent losses. All others (besides the core losses, which are also entirely independent) are partly independent and proportional (to the independent wall and core losses).

The heat carried away by the cooling water of the reactor, transformer, busses, and electrode holders must not be entered into this energy balance as losses. The losses which cause the temperature rise of the water occur in any case, and the water serves merely to prevent an excessive temperature rise. The losses determined by electrical measurements have nothing to do with the cooling water. On the other hand, the cooling of any mechanical part on the furnace shell, *e. g.*, doors, is a straight heat loss, increasing the total necessary heat in the furnace body. In case of series connection of the water (passing first through the electrical parts and then through the door frames, etc.), the elevated initial

temperature of the water when it reaches the furnace shell must be taken into consideration.

For comparison, Figure 145 is given, showing a conventional diagram for the energy balance. The total energy input of 100% is broken down into its various components. Unfortunately, this diagram holds for only one load. That of Figure 144 clearly indicates the relationship between the various losses.

Bettering the efficiency of the electrodes not only causes direct savings in the electrode losses but also decreases the losses in the busses, in the transformer, and in the reactor. Similarly, an increase (or decrease) of the wall losses of, say, a kwhr (over the period of comparison of perhaps 24 hr) affects the total energy consumption for a much larger amount than a. The energy consumption can also be expressed by an equation. Let U_F and L_F designate the useful energy and the heat losses in the furnace proper, η the efficiency in general, the subscripts E, B, TS, TP, and R the electrodes, busses, transformer secondary, transformer primary and reactor, respectively, L_C the core losses, and Q_{total} the total energy on the primary of the transformer. Then:

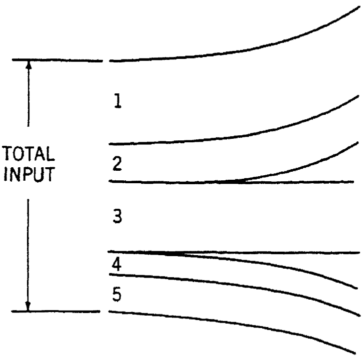


FIG. 145.—Conventional energy balance: 1, useful heat; 2, wall losses; 3, electrode losses; 4, losses in busses; 5, losses in transformer and reactor.

$$Q_{total} = \left[(U_F + L_F)\frac{1}{\eta_E\eta_B\eta_{TS}} + L_C\right]\frac{1}{\eta_{TP}\eta_R} \tag{79}$$

This can also be written as follows:

$$Q_{total} = (U_F + L_F + \eta_E\eta_B\eta_{TS}L_C)\frac{1}{\eta_E\eta_B\eta_{TS}\eta_{TP}\eta_R} \tag{79a}$$

All the efficiency factors can now be combined as follows:

$$\eta_{el} = \eta_E\eta_B\eta_{TB}\eta_{TP}\eta_R \tag{80}$$

Hence:

$$Q_{total} = (U_F + L_F + L_C\eta_E\eta_B\eta_{TS})\frac{1}{\eta_{el}} \tag{79b}$$

This equation thus indicates that it is wrong to speak of thermal efficiency in the same way as of electric efficiency and that change in the electric losses or in the heat losses has a far greater influence than would be apparent from the accompanying change in kwhr or Btu of that loss item.

Equation (79b) lends itself to another very telling way of presentation. Select a scale of in per kw. In Figure 146, U_F is plotted in this scale as abscissa to the right of the origin, O (line OA), L_F to the left (line OE). At point E, draw an angle α_1 characterized by

$$\cos \alpha_1 = \eta_E \eta_B \eta_{TS}$$

Mark the length of L_C on the side of the angle (EF); drop a perpendicular (FB) from F to the abscissa. Then line EB is:

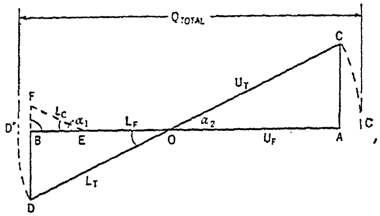


Fig. 146.—Energy balance (trigonometric method).

$$EB = L_C \cdot \eta_E \eta_B \eta_{TS}$$

and the distance OB is:

$$OB = L_F + L_C \cdot \eta_E \eta_B \eta_{TS}$$

Now draw an angle α_2 through the origin, the angle characterized by:

$$\cos \alpha_2 = \eta_{el}$$

Then erect perpendiculars at A (AC) and B (BD). The length U_T is (in the selected scale) proportional to the useful energy plus the proportional losses (the electric losses caused by the flow of useful energy). The length L_T is proportional to the sum of the independent losses and nonproportional dependent losses (the wall losses plus the electric losses necessary to supply the energy used for covering the wall losses).

By drawing two arcs, C' and D' are marked on the abscissa axis; $L_T + U_T$ is proportional to the total energy input, Q_{total}. From this presentation it again becomes obvious that neither the value Q_{total}, nor an over-all efficiency figure is sufficient to characterize the power consumption of a furnace. If L_F or L_C increases, the increase of Q_{total} is larger than that of L_F or L_C.

L. Consequences

The most important deduction to be made from the above discussions is the uselessness of the average energy balances. Since no better balances have been published so far, some traditional balances will be discussed and analyzed in Section M following.

The fact that the electric losses are in part dependent (on heat losses) and in part proportional (to the useful load) raises the question of which, if any, type of loss should be preferred, in order to obtain best over-all efficiency. It is worth considering for a moment the possibility of lengthening the melting time by decreasing the connected load: the useful power drops and the electric efficiency increases. However a brief analysis shows that this never pays from the viewpoint of energy losses. The total useful power should be as high as possible as long as the charge

is capable of absorbing the heat. In some instances, there might be a problem of electric *vs.* heat losses, although it is hard to foresee how this can occur. If increase of the one and decrease of the other are of the same percentage, it is again always desirable to keep the electric efficiency as high as possible, because η_{el} causes losses due to the heat losses as well as due to the useful heat. If, however, the changes of the two losses (in opposite direction) are not in the same degree, individual analysis must be employed to discover which combination yields the lowest losses.

M. Actual Balances

The energy balances published in Europe refer mostly to furnaces of small size, up to eight tons, and only occasionally up to 15 tons. Lyche

TABLE XX

ENERGY BALANCE OF A SMALL FURNACE (AFTER LYCHE AND NEUHAUS)

A. Heat Input

Heat used	Melting period, kwhr	Refining period, kwhr	Total kwhr	Per cent of total heat input
Electric energy input	3350	2200	5550	84.81
Heat content of charge	20	3	23	0.35
Heat of reaction	193	317	510	7.79
Heat produced by burning off of electrodes	243	218	461	7.05
TOTAL HEAT INPUT:			6544	100.00

B. Heat Expenditures

Heat lost	Charging period, kwhr	Melting period, kwhr	Refining period, kwhr	Total kwhr	Per cent of total heat expenditure
Heat content of steel		2543	168	2711	41.46
Heat content of slag		117	448	565	8.65
Coking and baking of electrodes		1	1	2	0.03
Wall losses	137	445	516	1098	16.78
Losses through escaping gases		97	206	303	4.62
Losses through water cooling	92	340	399	83	12.69
Electric losses		518	457	975	14.87
Losses during charging (found as difference)	59			59	0.90
TOTAL HEAT EXPENDITURE:				6544	100.00

and Neuhaus[89] report on the measurements on a six-ton furnace equipped with a 200-kva transformer. The maximum voltage was 173 v. The melting time was 3 hr 22 min, the refining time 3 hr 31 min. The furnace

[89] L. Lyche and H. Neuhaus, *Ber. Stahlwerksausschusses Eisenhüttenleute,* **101** (1926); abstracted in *Stahl u. Eisen,* **46**, 780 (1926).

was operated with self-baking Söderberg electrodes. The charge was 16,500 lb. The balance as presented by the authors is shown in Table XX. Besides the general shortcoming in not separating the proportional and the dependent losses, the balance shows the typical error of adding the electric losses to the losses by water cooling. The latter are composed of the loss in the electrode rings, door frame, roof ring, etc. The heat losses in the electrode rings and the electrode holders are, at least in part, electrical losses and therefore are added twice.

Similarly, Mueller,[90] reporting on the difference between an energy balance with a new lining and one with worn-out lining, gives the electric losses and in addition the losses from the outside of the electrodes, from cooling water, and from the electrode holders. (This balance refers to a six-ton furnace for smelting of ferro-manganese.)

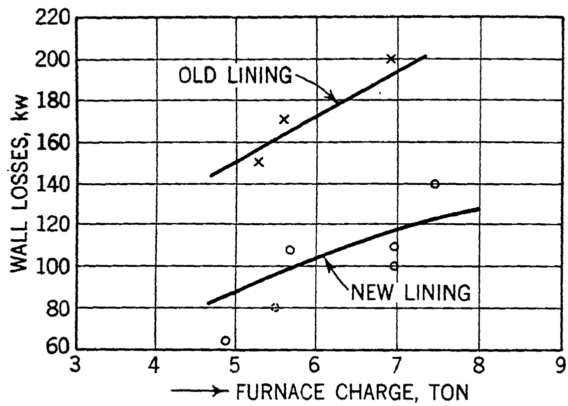


FIG. 147.—Influence of life of lining on energy balance (after Kriz[91]).

Kriz [91] analyzes a large number of furnaces. A brief abstract of his balance is presented here. On an average the losses in the busses and electrodes amount to 6% of the primary input of the transformer, this sum not including the losses of the transformer proper. The cooling water causes losses of 35 kw in five-ton furnaces and 60 kw in seven-ton furnaces. The losses through door opening amount in general to 50–80 kw in furnaces of 5–7 tons. Escaping gases in these furnaces are estimated at 30–60 kw. Ess (*loc. cit.*, page 72) estimates the losses from escaping gases to be 30–120 kw. The losses from wall and roof depend largely on the age of the lining. Kriz found the averages shown in Figure 147.

[90] E. Mueller, *Stahl u. Eisen*, 59, 126 (1939).

[91] S. Kriz. *Arch. Eisenhüttenw.*, 1, 413 (1927).

Widdel[92] finds for a six-ton furnace—new at the time—an over-all efficiency of some 60%. Of the roughly 40% losses, 11.4% are classified as wall losses, 10.1% as electric losses, 8.9% as due to cooling water, 7.5% in the escaping gases, and 1.5% unaccounted. The figures refer to a total consumption of 5339 kwh. Melting took 2 hr 20 min, refining 1 hr 45 min. The transformer was rated at 1600 kva, to be overloaded to 2000 kva. The electric energy is credited with only 88.1% of the heat income (at 90% during the melting period, 78.5% during the refining period), while the balance is credited to various chemical reactions, including burning off of electrodes (1.1%) and oxidation of silicon during the refining period (11.9%).

Wark,[93] analyzing a seven-ton furnace, arrives, by and large, at similar results: the over-all efficiency as found by him is in the order of magnitude of 45–50%. Inasmuch as times of melting and refining are not shown for all charges, comparisons are hard to make and the value of the paper is greatly diminished.

VIII. FERRO-ALLOY FURNACES

A. Introduction

Although this discussion refers particularly to ferro-alloy furnaces, by and large the statements apply also to other furnaces for electrochemical purposes, as, for example, furnaces for phosphates, etc.

Ferro-alloy furnaces are distinguished from steel melting furnaces mainly in two aspects. First, they are mostly pure resistance furnaces. Formation of an arc is in most cases undesirable and should be avoided. But even if part of the energy is transformed in an arc component, the resistance component is important. In the steel melting furnace, the energy is transformed predominantly in the arc. Second, the charge in the ferro-alloy furnace undergoes much more severe changes than the charge in the steel furnace. The chemical composition of the furnace content at various levels above the hearth is almost uniform in steel furnaces (except for the slag cover), and is entirely different in ferro-alloy furnaces.

With the energy transformation taking place in the charge in ferro-alloy furnaces, a number of problems occur which are not encountered in the steel melting furnace. Also, certain aspects of ferro-alloy furnace design are less readily analyzed than those of steel furnaces, while other parts of the design follow the same analysis as that for steel furnaces.

In the following section the more important points in which ferro-alloy furnaces differ from steel furnaces will be discussed. Briefly, the

[92] E. Widdel, *Stahl u. Eisen,* **53,** 1265 (1933).
[93] N. Wark, *Arch. Eisenhüttenw.,* **2,** 145 (1928).

designs of the furnace body differ widely while those of busses and transformers differ only slightly.

In the calculation, the main difficulty is encountered in defining the useful resistance of the load. Connected herewith are difficulties in electrode design, which is perhaps still more empirical than in arc furnaces. Wherever complete rational analysis is impossible, an attempt will be made at least to outline qualitatively the problems involved. The importance of line voltage fluctuations is considerably greater than in steel furnaces.

B. Furnace Body and Lining—Number of Phases

Ferro-alloy furnaces are usually nontiltable. They are charged continuously from the top, and discharged at relatively short intervals of from one to three hours by tapping. In normal operation they are never emptied entirely; rather, at each tapping, only a fraction of the content is removed.

Being stationary, the furnace body is in general simpler in mechanical design than that of steel furnaces; in contrast to the latter, ferro-alloy furnaces are usually "home-built," their design varying considerably from plant to plant.

In most cases the furnace is "open." No bricked roof is applied. The actual hot zone is covered and protected by the charge which is put in from the top. Gases forming because of the chemical reactions which take place escape through the spaces within the fresh charge and, as far as they are combustible, burn at the top of the furnace, with luminous flames. Thus, a ferro-alloy furnace in operation is much less clean and neat than a steel furnace.

The inside furnace lining is usually made of carbon, with some other materials used occasionally. For calcium carbide, and for alloys such as ferrochrome or low-carbon ferromanganese which have an affinity for carbon, burned magnesite or the ore of the metal which is being produced is used. Carbon is so good a conductor of heat that it must be backed by refractory material, which acts as a heat insulator. Sometimes an outside layer of real insulating material must be used. In designing the wall, special care must be taken to avoid on the inside surface temperatures which are beyond the safe operating limit of the inside lining. (See page 208 for calculation of temperature drop in walls.) Figure 148 illustrates a typical lining of a ferro-alloy furnace.

Although arc furnaces for the melting of steel (with the exception of the rocking arc furnace) are today almost always built as three-phase furnaces, ferro-alloy furnaces are sometimes built as single-phase or double-phase furnaces. The shape of the furnace body is different for different furnaces.

Single-phase furnaces can have one or two electrodes. One-electrode furnaces with current return through the bottom are simplest from an electrical point of view: there is only one path for the current to follow, and one reaction zone. Such furnaces are generally cylindrical, with one round electrode in the center. These furnaces are preferred for small units or for products whose manufacture is delicate. The return current is carried away from the bottom of the furnace by various methods, two

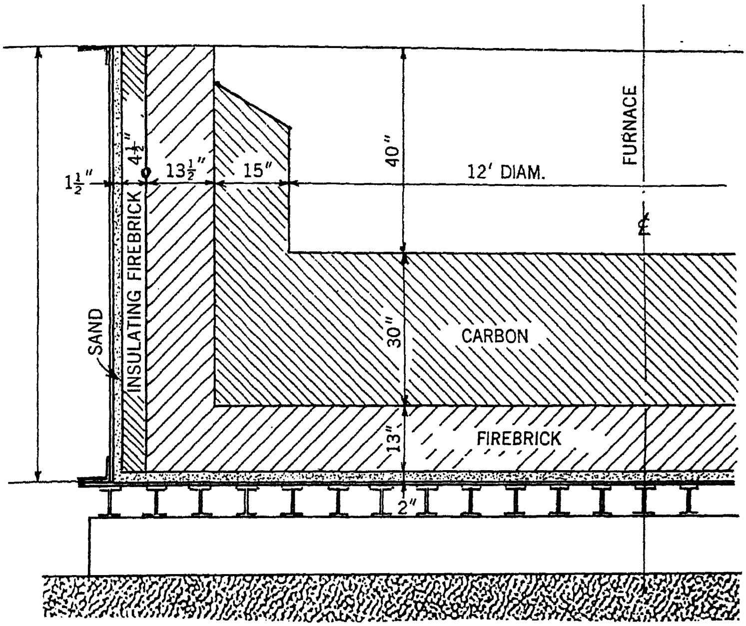


FIG. 148.—Typical lining of a ferro-alloy furnace.

of which are shown in Figure 149. A copper section must be inserted in the shell to cut the magnetic circuit unless the return is inside the shell, as in the Miguet furnace mentioned below. In arrangement *I*, the carbon is brought out and connected to a copper bus outside the furnace; in arrangement *II*, water-cooled copper tubes are brought into the furnace.

In France, single-electrode furnaces of 3500 kw were standard for many decades. One of the best perfected designs is the Miguet-Perron furnace, which has been built for as much as 15,000 kw, 300,000 amp. Such large furnaces would unbalance ordinary three-phase systems and are therefore generally not acceptable in the United States. They are successful in those relatively small plants, with shop-owned power supply, in which each furnace is fed by an individual single-phase generator. (Such generators in small hydroelectric plants are frequently encountered

in the French Alps, where the ferro-alloy industry is rather decentralized in small units.)

Two-electrode furnaces are not quite so simple electrically. Although the same current flows in both electrodes, the voltage is not necessarily the same at the tips of the two electrodes. These furnaces are rectangular with round or square ends.

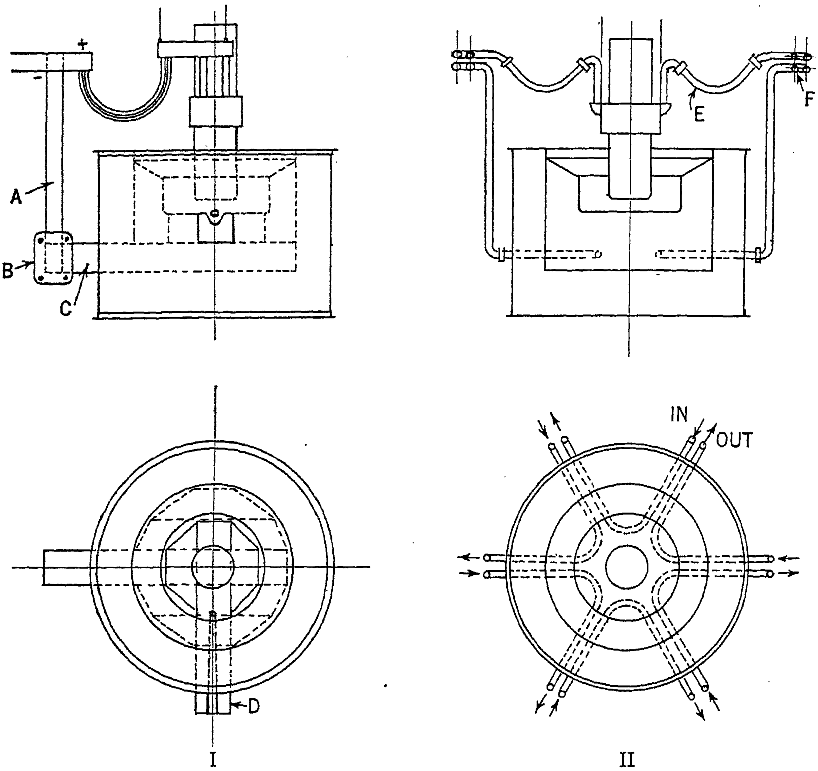


FIG. 149.—Typical connections of bottom electrodes. *I*, carbon-connection: *A*, copper bus; *B*, clamp; *C*, carbon rod; *D*, spout. *II*, water-cooled pipes (water-cooled copper pipes embedded in carbon paste); *E*, water-cooled flexible cable; *F*, ring bus.

Two-phase furnaces are seldom used. If fed from a three-phase system, they require special Scott connected transformers. They have two electrodes with return through the bottom.

The majority of ferro-alloy furnaces in this country are of the three-phase type. Three-phase furnaces are built with three or six electrodes. The electrodes can be arranged in a straight line or in the corners of a triangle (three electrodes) or in the corners of a hexagon (six electrodes). The electrodes in line permit the use of rectangular furnaces which are mechanically simple.

The logical arrangement for the electrodes of a three-phase furnace is in a triangle, permitting a complete balancing of the phases and resulting also in a much lower reactance. The furnaces in this case are round or clover-leaf shaped. The round furnace is simpler; but the triangular furnace requires less material, and can be designed with a smaller reactance because the interlaced bus can be brought closer to the electrodes.

C. Size and Shape of Furnace Body

In steel melting furnaces the greatest part of the heat is generated above or at the level of the charge. Heat transfer thence occurs by conduction and (in the liquid bath) by convection. The size and shape of the furnace body are determined by the possibility of distributing heat evenly. In the case of ferro-alloy furnaces, conditions are much more complicated: The electric resistance of the charge before the chemical reaction takes place is of a higher order of magnitude than the resistance of the finished ferro-alloy; the current must pass from one electrode to the next; over the entire length of the path the electric resistivity and the cross section of the path change; and an increase in size of furnace and a change of shape evidently influence the characteristics of the path, and therewith the electric behavior of the furnace. These considerations tie in closely with the design of the electrodes.

The entire field is not yet amenable to rational approach. Some qualitative considerations follow:

1. Single-Phase Furnace with One Central Electrode

The furnace is schematically represented in Figure 150. In this figure, the three dimensions important in connection with the present considerations are marked by capital letters: D_1 is the electrode diameter; D_2 is the crucible diameter; and H is the distance between electrode tip and hearth. The lower case letters describe the various parts, as per explanation in the caption. The operation of the furnace does not change materially whether the carbon sides extend to the top of the sidewalls (c plus d) or only part way up (c). The resistance of the charge, h, facing d is so high that no appreciable amount of current can pass there. The liquid ferro-alloy, j, accumulates for tapping, its level (interface g–j) being different before and after tapping. The shape of zone f of most intensive reaction is not known. It depends, undoubtedly, on the values of D_1, D_2 and H_1, as well as on the current density of the electrode. From any point, m, on the perimeter of the tip of the electrode, the current flows in two directions: toward the bottom (b) and the sides (c). The current path toward the bottom has constant cross section; the energy production at various levels changes only with the resistivity of the material. The flow towards sides c, however, is a radial cylindrical

flow, having at its disposal increasing cross section. If the resistivity would not change, the flow could be calculated by Equation (13) on page 37, which gives the thermal flow through cylindrical shells. With decreasing resistance, the energy production toward the outside becomes smaller. Consequently, the temperature is lower on the outside; this in turn causes a higher resistivity near the outside, which in part counteracts the greater available area.

The ratio, D_2/D_1, determines the electrical resistance for radial flow (assuming constant resistivity). If D_2/D_1 becomes too small, an exces-

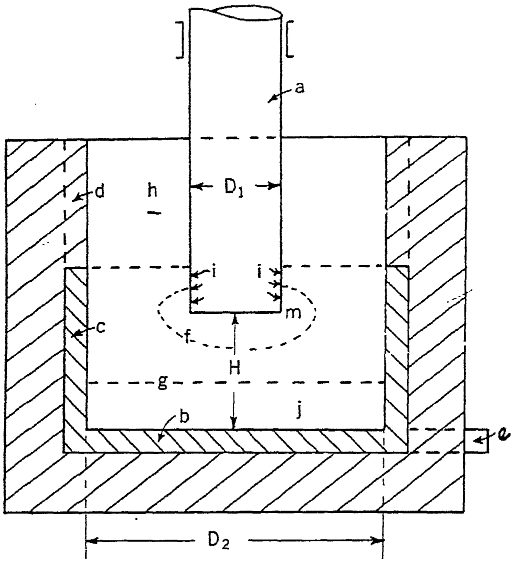


FIG. 150.—Current flow from electrode: *a*, electrode; *b*, carbon bottom; *c*, *d*, sides of crucible; *e*, connector for carbon bottom; *f*, *g*, *h*, parts of the charge.

sive amount of current flows to the sides, and the sides burn out. As long as there is current flowing to the sidewalls, the electrode also conducts current on its sides, as per arrows *i*; in addition, of course, current flows from the bottom surface of the electrode to the bottom (*b*) of the lining. As this ratio increases, the current, part *i*, becomes smaller and smaller, and at some value of D_2/D_1 there will be almost no current flowing from the electrode sides. When such a ratio is reached, the walls (*c*) are protected by a layer of unmolten ore. The entire flow is directed downward.

The result of such design is a saving in life of the sidewall, but probably a decrease in electrode life. Almost the entire electrical energy

is now forced through the bottom. A higher electric load in such a case can be achieved only by increasing D_1 (and then of course also D_2 in order to maintain the desired ratio).

The actual values of D_2/D_1 at which no lateral flow occurs depend on the material in question and on the value of H; the greater H is, the higher D_2/D_1 must be to avoid lateral flow. (It should be understood that "no lateral flow" must not be taken literally: very near the bottom of the electrode some current will leave the electrode sideways; the current lines then curve downward because of the high resistance of outside layers. This in turn limits to a short length the zone of lateral flow of the current out of the electrode. See Figure 151.)

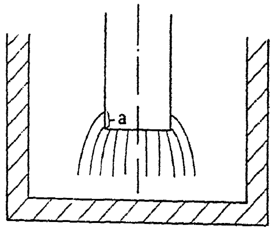


FIG. 151.—Energy flow from electrode (lateral flow limited to short zone a).

Conclusive proof as to which method is better is not available. Practice seems to favor the design with purely axial flow toward the bottom.

2. Single-Phase Two-Electrode Furnace

If the electrodes are placed too close together, current will flow from one electrode to the other rather than from either electrode to the bottom. Conditions would be even worse than in the case of the single-electrode furnace with small ratio D_2/D_1; in the case of the two-electrode furnace, the electrodes would burn off unevenly. Therefore, for such furnaces, it is desirable to space the electrodes so far apart that current flows only to (and from) the bottom.

3. Three-Phase Furnace

If the electrodes are in a straight line (rectangular furnace body) the same considerations hold as in the explanation for the single-phase two-electrode furnace. The electrodes should be placed sufficiently far apart to safeguard against the flow of current from one electrode directly to the next. The furnace should really consist of a number of single-phase units, each being connected to a different phase on one side and to a common neutral on the other side.

One could even go so far as to build little baffle walls between the electrodes (Fig. 152). Electrically, this arrangement is not so satisfactory because of the asymmetry and the resulting unbalance of the phases (see page 184). Moreover the reactance of such furnaces is relatively high and increases with the size of the furnace. Since the charge in the furnace itself forms the major part of the resistance, the resistance decreases with increasing size of the furnace. Thus the power factor

decreases and becomes unsatisfactory at a furnace size of more than 4000 kw. Figure 153 illustrates these conditions. In the case of six electrodes in a row, the arrangement shown in Figure 154 results in equal potential between any two adjacent electrodes.

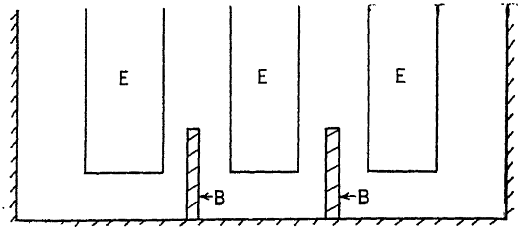


FIG. 152.—Baffle plates for three-electrode furnace: E, electrodes; B, baffle plates.

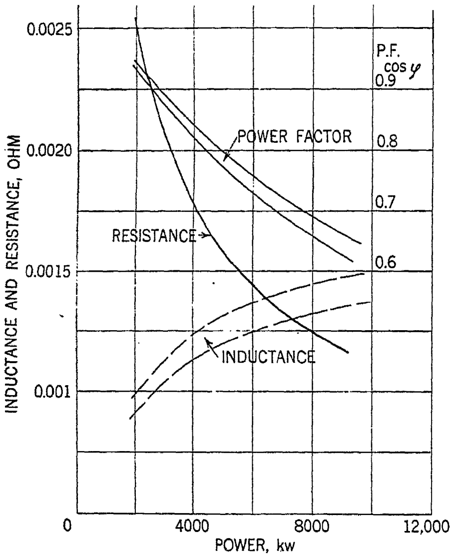


FIG. 153.—Resistance, inductance, and power factor *vs.* power. The two lines indicate the upper and lower limits of inductance and power factor.

D. ELECTRODES

In parts of the charge which do not conduct, the electrode embedded therein does not give off current to the charge. However, heat is generated within the electrode, and this heat, at least in part, is given off to the charge. Thus, for any element of the electrode (Fig. 155) the following heat balance can be written:

$$Q_i + Q_g = Q_o + Q_L$$

G. Characteristic Functions

Previously, diagrams based on characteristic figures and current diagrams were discussed (page 167 and 185). For ferro-alloy furnaces, another type of diagram (Fig. 158) is frequently used which could, of course, also be applied to steel melting furnaces.

By way of example, Figure 158 is drawn for an inductance of 0.0009 ohm per phase, and a 30-in carbon electrode. The ordinates represent the power input; the abscissas represent the resistance, including load and furnace leads, and, since the reactance is constant, the corresponding power factor. The straight lines rising toward the right are the electrode amperes. The low tension voltage taps are shown as curved lines falling toward the right. For any chosen point on the chart, all the important functions can be immediately determined.

The chart shows the limits between which the furnace will operate satisfactorily. These are: the contract power factor; the maximum current which the electrodes will stand; the highest and lowest taps; the highest resistance at which the furnace will operate satisfactorily; and the capacity of the transformer.

It will be apparent why the transformer taps should not be too far apart. Suppose it is desired to operate the furnace at 5500 kw. On the 110-v tap, the current would be about 32,500 amp, the power factor around 0.89, the resistance 0.00174 ohm. On the 105-v tap, the current would be 35,600 amp, the power factor 0.85, and the resistance 0.00145 ohm. While the resistance is a little high on the 110-v tap, which will make it difficult to keep the electrodes down, on the 105-v tap the current is above the safe limit for a 30-in electrode. If the taps were farther apart, it could very well happen that neither of the two would be suitable.

H. Design Based on Similarity

Calculation of electrode melting furnaces is always lengthy. Many factors are not as yet known accurately, and cannot be determined beforehand. It is therefore desirable to investigate the manner in which experience acquired from an existing furnace can be applied to the design of a new furnace. If data are available for one furnace in operation, the relationship of resistance, voltages, power, and current to the electrode diameter can readily be determined. A set of relations has been developed which has been found very convenient in such cases.

A study of several furnaces of different sizes working on the same product with the same raw materials seems to show that, for satisfactory conditions, the product of the resistance in the furnace times the diameter of the electrode varies within narrow limits. This means that the path of the current between the electrode and the furnace bottom has a constant resistance for a certain fixed segment of electrode periphery; it is made up of a certain number of parallel circuits of the same width and

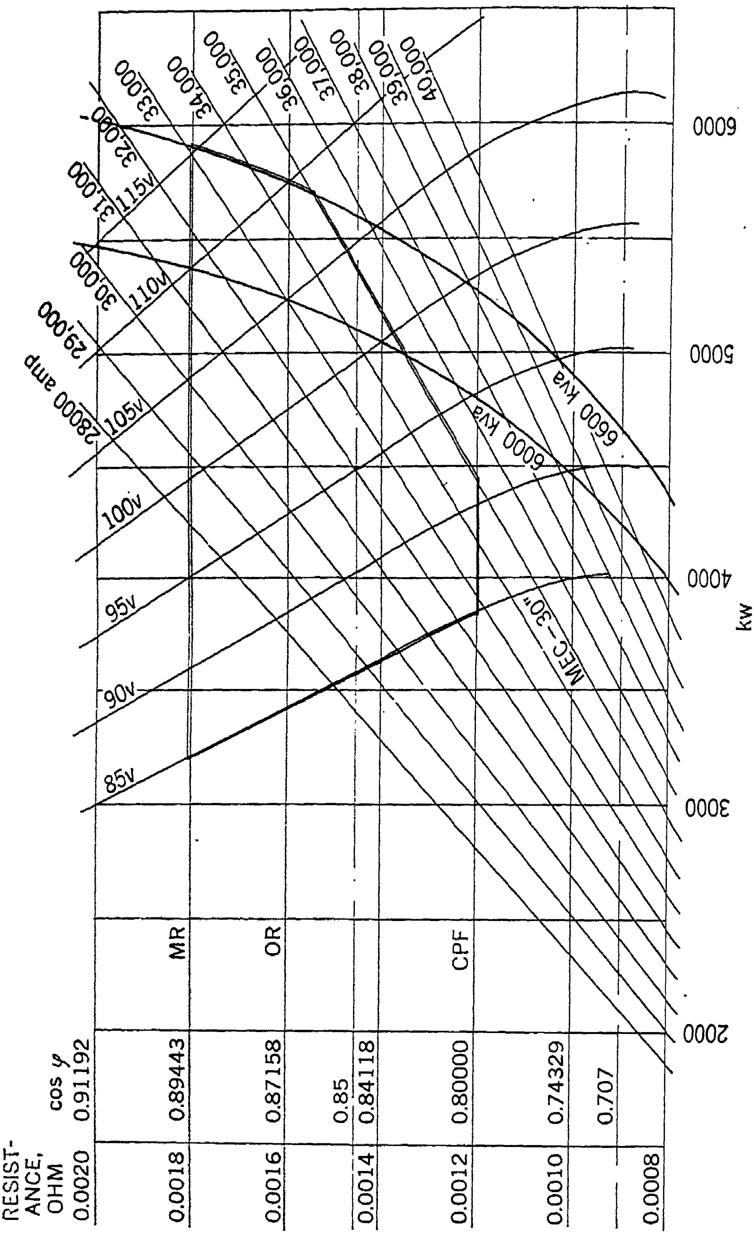


FIG. 158.—Example of operating diagram of a ferro-alloy furnace. *MR*, maximum operating resistance; *OR*, optimum operating resistance; *CPF*, contract power factor; *MEC*, maximum electrode current (30-in electrode).

resistance, the number varying in direct proportion with the periphery (or diameter) of the electrode. This relation is expressed by the formula:

$$RD = C \tag{81}$$

The value of C varies with the product, and for the same product with the raw materials used; D is the electrode diameter in inches; and R is the furnace resistance in ohm per phase.

It can now be shown that, for normal conditions of electrode operation, the current should be changed with the (3/2) power of the electrode diameter:

I_1

$$\frac{I}{I_1} = \left(\frac{D}{D_1}\right)^{3/2} \tag{82}$$

PROOF

Assuming that generated and radiated heat are balanced, which is practically the case in most furnaces, it is possible to establish a relation between current and electrode diameter. If the surface temperature is t_s and t_a is the air temperature, then the radiated heat, Q_r, per foot of electrode length is equal to:

$$Q_r = 12 \times h(t_s - t_a)\frac{\pi D}{12} \text{ watt per ft}$$

where h is the boundary conductance in watt per sq in, C.

The heat generated per foot of electrode is equal to:

$$Q_r = \frac{4}{\pi}\rho\frac{I^2}{D^2} \times 12 \tag{83}$$

where ρ is the resistivity of the electrode in ohm $\times$ inch. Hence:

$$h(t_s - t_a)\frac{\pi^2 D^3}{4} = I^2\rho$$

or, with:

$$\frac{h(t_s - t_a)\pi^2}{4 \times \rho} = C$$

$$CD^3 = I^2 \tag{84}$$

Now, for a different I current, C remains unchanged. Hence:

$$\frac{I}{I_1} = \left(\frac{D}{D_1}\right)^{3/2}$$

By combining Equations (81) and (82), it is easily found that:

$$\frac{W}{W_1} = \left(\frac{D}{D_1}\right)^2 \tag{85}$$

$$\frac{E}{E_1} = \sqrt{\frac{D}{D_1}} \tag{86}$$

From these equations it may readily be seen that the power densities (watt per sq in) in the electrode are constant, if assumptions for Equations (81) and (82) stand.

I. Line Voltage Fluctuations

Line voltage fluctuations are more important in the operation of ferro-alloy furnaces than of steel furnaces. The phenomena are the same, but the larger size and load of the alloy furnaces make the results of fluctuations numerically more important. The entire operation of a furnace depends on the voltage. If the substation voltage drops more than can be adjusted by voltage taps on the furnace transformer, the power absorbed by the furnace is bound to decrease. Other additional difficulties arise. The undesirable consequences of line voltage fluctuation are: (1) operating difficulties; (2) difficulties of control; and (3) difficulties with power contract.

1. Operating Difficulties

The furnace operator should, generally speaking, maintain as constant a load as possible. So long as the voltage is constant, it is sufficient to keep the current constant. If the voltage drops the current drops also. The natural reaction of the operator will be to increase the current by lowering the electrode. Conditions in practice are worse. The operator will, in most cases, attempt to keep the total power constant (rather than the useful power—if the latter were attempted the current settings would have to be changed as the voltage changes). As the plant voltage goes down, the current must be increased (by lowering the electrode) In so doing, the power factor deteriorates and the useful power drops. If the voltage then increases later, the furnace, operating at a low power factor and with a low resistance, r_a, of the load, does not return to the same operating conditions.

The voltage drop in the lines from the substation to the furnace aggravates conditions. An example will help to illustrate matters:

Assume that the total power of a three-phase furnace is 17,500 kw. The normal plant voltage is 12,500 v and the normal power factor, 0.85. The furnace reactance (leads + transformer) is 3.998 ohm (referring to the high voltage side); the line from the substation to the furnace transformer has a resistance of 0.1822 ohm and a reactance of 0.6854 ohm. The voltage at the substation is normally 13306.6 (resulting from the furnace voltage, current and power factor and line properties). Now assume that the voltage at the substation increases to 13,700 v, then decreases, and continues to decrease until 12,985 v (below this voltage the power of 17,500 kw is no longer obtainable in the furnace). Then the voltage increases again. In increasing, it does not reach its previous value. Table XXI illustrates the conditions. The solution would be to give the operator a diagram indicating the desirable current for

each voltage. This desirable current should be calculated for the maximum useful power (see page 171).

2. Difficulties in Control

Control based on total power is unsatisfactory as proved by Table XXI. Control based on constant resistance of the bath (yielding equilibrium only if the current changes in the same proportion as the voltage) is unsatisfactory because it causes a decrease of furnace power (total power as well as useful power) at a higher rate than is indicated as necessary by the operating diagram. The ideal control would be based on useful power, with provision made for automatically reducing the power setting if the electrode comes too close to the bottom (because of low voltage and consequent need for low bath resistance). In view of the practical impossibility of measuring the useful power, control based on the total power, automatically adjusted if the voltage changes, would be the most desirable solution.

TABLE XXI

CONDITIONS FOR CONSTANT POWER OF 17,500 KW DELIVERED TO PLANT

Substation voltage	Current	Power factor	Plant voltage	Plant power factor	Plant resistance	Kw losses in furnace leads [a]	Useful power in furnace
13700	878.7	0.8540	13010	0.8839	7.555	896.7	16603
13600	874.1	0.8540	12884	0.8770	7.296	928.4	16572
13500	911.2	0.8368	12757	0.8691	7.025	964.2	16534
13400	930.4	0.8263	12626	0.8600	6.739	1005.1	16495
13306.6	950.3	0.8148	12500	0.8500	6.451	1048.6	16451[b]
13200	979.2	0.7987	12347	0.8357	6.084	1113.2	16387
13100	1014.2	0.7782	12190	0.8173	5.671	1194.3	16306
13000	1078.2	0.7399	11983	0.7821	5.019	1349.4	16151
12987.7	1132.1	0.7071	11879	0.7513	4.551	1488.1	16012
13000	1154.8	0.6933	11853	0.7381	4.374	1548.3	15952
13064.4	1207.9	0.6614	11820	0.7071	3.998	1694.1	15806
13100	1227.6	0.6497	11832	0.6956	3.871	1749.7	15750
13200	1271.6	0.6240	11860	0.6700	3.608	1877.3	15623
13306.6	1309.3	0.6025	11906	0.6382	3.403	1990.4	15519
13400	1338.1	0.5863	11953	0.6317	3.258	2079.0	15421
13500	1366.3	0.5709	11008	0.6158	3.125	2167.4	15333
13600	1392.4	0.5570	12067	0.6013	3.009	2250.9	15249
13700	1416.9	0.5442	12143	0.5872	2.905	2330.9	15169

[a] Including transformers, busses, and electrodes.
[b] Normal conditions.

3. Difficulties with Power Contract

Most power contracts have a demand charge based on a maximum peak over a predetermined period (half-hour or one hour). Some also have a power-factor clause penalizing the customer when the power factor drops below a given value. In most contracts, the power factor on which the calculations are based is taken at the time of the peak. While these

clauses can easily be kept when the voltage is constant, they cause co siderable difficulty when the voltage varies. If, when the voltage drop the furnace user tries to keep his load constant in order to maintain h demand charge within reasonable limits, he will probably run into ve low power factors. If the monthly peak occurs at a time when the vo age is low, the power factor will be very unfavorable. The bill will l based on the poor power factor; the furnace user is thus penalized f something which is entirely out of his control. Suppose that, in th example, a peak of 17,500 kw occurred when the voltage at the substatio was 13,400 v; the power factor would be 0.8600. If the same peak 17,500 kw occurred when the voltage was 13,000 v, the power factor wou be 0.7821.

In periods of high voltage, the resistance of the furnace would hav to be increased considerably (raising the electrodes) in order to keep th power constant. In fact, there is even the danger that the electrode ma leave the furnace if raised too much. To avoid this, the operator w risk an overload. In order not to exceed his demand charge, he ma then shut down the furnace for the balance of the measuring peric (15 min, etc.). Cutting out so heavy a load as a furnace of course caus considerable disturbance in the system and in other plants.

These difficulties can be avoided either if the power company induc the user to keep the furnace going continuously by more liberal contrac or if a voltage regulator is installed which gives constant voltage ind pendent of line voltage fluctuations. Such "liberal contracts" ma include longer demand periods, averaging the power factor over an enti month or adjusting the maximum nonpenalized load and/or power fact to the minimum line voltage at the time of measurement.

APPENDIX

A. Thermal Conductivities[a]

Material	Temperature, degrees F	Conductivity, Btu/ft, hr, F
Aluminum	212	119
Brass (70 copper, 30 zinc)	212	60
Cast iron	212	30
Silver	212	238
Steel, mild	212	26
Zinc	212	64
Asbestos	32	0.087
	212	0.111
	392	0.120
	752	0.129
Aluminum foil, 7 air spaces per 2.5 in.	100	0.025
Bricks		
Alumina (92–99% Al_2O_3 by weight), fused	801	1.8
Chrome brick (32% Cr_2O_3 by weight)	392	0.67
Magnesite (86.8% MgO, 6.3% Fe_2O_3, 3% CaO, 2.6% SiO_2 by weight)	399	2.2
Silicon carbide brick, recrystallized	1112	10.7
Fiber insulating board	70	0.028
Lava	....	0.49
Mineral wool	86	0.0225
Slag, blast furnace	75–261	0.064

B. Emissivities[a]

(When two temperatures and two emissivities are given, they correspond, first to first and second to second, and linear interpolation is permissible.)

Material	Temperature, degrees F	Emissivity
Aluminum		
Highly polished plate, 98.3% pure	440–1070	0.039–0.057
Oxidized at 1110 F	390–1110	0.11–0.19
Brass		
Highly polished, 73.2% Cu, 26.7% Zn	476–674	0.028–0.031
Polished	100–600	0.096–0.096
Dull plate	120–660	0.22
Oxidized by heating at 1110 F	390–1110	0.61–0.59
opper		
Polished	242	0.023
Plate heated at 1110 F	390–1110	0.57–0.57
Molten copper	1970–2330	0.16–0.13

.bstracted from W. H. McAdams, *Heat Transmission*. McGraw-Hill, New York, 1942.

B. Emissivities—*Contd.*

Material	Temperature, degrees F	Emissivity
Iron and steel		
Metallic surfaces (or very thin oxide layer)		
Polished iron	800–1880	0.144–0.3
Polished steel casting	1420–1900	0.52–0.5[illegible]
Oxidized surfaces		
Rolled sheet steel	70	0.657
Oxidized iron	212	0.736
Steel, oxidized at 1110 F	390–1110	0.79–0.79
Rough ingot iron	1700–2040	0.87–0.95
Cast plate, rough	73	0.82
Cast iron, rough, strongly oxidized	100–480	0.95
Molten metal		
Cast iron	2370–2550	0.29–0.29
Lead		
Gray, oxidized	75	0.281
Oxidized at 390 F	390	0.63
Nickel		
Plate, oxidized by heating at 1110 F	390–1110	0.37–0.48
Alloys		
Chromnickel	125–1894	0.64–0.76
NCT-3 alloy (20 Ni, 25 Cr), brown splotched, oxidized from service	420–980	0.90–0.97
Zinc		
Oxidized by heating at 750 F	750	0.11
Brick		
Silica, unglazed, rough	1832	0.80
Carbon		
T-carbon (Gebruder Siemens), 0.9% ash	260–1160	0.81–0.79
Refractory materials, 40 different	1110–1830	
Poor radiators		0.65–0.70; 0.75
Good radiators		0.80–0.85; 0.85–0.90

C. Specific Heat and Heat of Fusion

Material	Temperature range	Specific heat, Btu/lb, F	Heat of fusion, Btu/lb	Approximate value of useful heat (from cold to melting point—no superheat), kwhr/lb
Steel, plain carbon	70–2300	0.168	126	0.154
Steel, molten, plain carbon	2500	0.200	—	—
Copper	70–1950	0.094	77.5	0.084
Brass	70–1800	0.092	66.5	—
Aluminum	70–1180	0.22	168	0.134
Zinc	70–750	0.094	50.5	0.0363
Tin	70–480	0.056	25	0.0146

SUBJECT INDEX